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The Three-body Problem in Riemannian Geometry. Hidden Irreversibility of the Classical Dynamical System

A. S. Gevorkyan

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Abstract

The classical three-body problem is formulated as a problem of geodesic flows on a Riemannian manifold. It is proved that a curved space allows to detect new hidden symmetries of the internal motion of a dynamical system and reduces the three-body problem to the system of 6*th* order. It is shown that the equivalence of the original Newtonian three-body problem and the developed representation provides coordinate transformations together with an underdetermined system of algebraic equations. The latter makes the system of geodesic equations relative to the evolution parameter (*internal time*), i.e. to the arc length of the geodesic curve, irreversible.

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Author information

Affiliations

1. Institute for Informatics and Automation Problems, National Academy of Sciences of the Republic of Armenia, Yerevan, 0014, Armenia

   A. S. Gevorkyan

2. Institute of Chemical Physics, National Academy of Sciences of the Republic of Armenia, Yerevan, 0014, Armenia

   A. S. Gevorkyan

Corresponding author

Correspondence to A. S. Gevorkyan.

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