International Conference Dedicated to 90th Anniversary of

SERGEY MERGELYAN

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Yerevan, 2018

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ABSTRACTS

Artificial and Biological Vision Synergistic Approach: Applications, Challenges and Perspectives

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The rapid proliferation of hand-held mobile computing devices, coupled with the acceleration of the Internet-of-Things connectivity, and data producing systems, such as embedded sensors, mobile phones, surveillance cameras, have certainly contributed to these advances. One of the fields in which scientific computing has made particular inroads has been the area of big image-data analytics and computational vision systems. In our modern digital information connected society, we are producing, storing and using ever-increasing volumes of the digital image and video content. Every day, we create 2.5 quintillion bytes of data (over 2.5 billion photos uploaded to Facebook every month, and over 300 hours of video uploaded to YouTube per minute by over 1 billion users) - so much that 90% of the data stored in the world today has been created in just the last two years. How can we possibly make sense of all this visual-centric data? And how can we be sure that the derived computations and analysis are fully relevant to our human vision, understanding and interpretations. Well managed and properly analyzed, this wealth of data can be used to unlock new sources of economic value and improved societal prosperity. The current state of the art in computational vision analytics provide us with a variety of tools and methods to solve various classes of computer vision problems. We then are posed with the following questions - how big of a class of problems in vision are we able currently to solve, compared with the totality of what humans can do? Can we duplicate human vision abilities in a computational device? In this talk, we will give an overview of the main areas of vision-based technology being investigated by Agaians visual computation and analysis research team. We will also discuss

- How do we render, interpret, and communicate all this data?
- What are the opportunities and challenges?
- What is the technological roadmap for the near future?

Generalized Pleijel Identity and Covariograms of Convex Bodies

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Let $\mathbb{D}$ be a convex bounded domain, S^1(=the circle of radius 1 centered at the origin) be the space of all directions in the plane and $[\mathbb{D}] = \{g \in \mathbb{G} : \chi(g) = g \cap \mathbb{D} \neq \emptyset\}$, where $\mathbb{G}$ is the space of lines in the plane. In the paper [1] generalized Pleijel identity for any locally-finite, bundleless measure in the space $\mathbf{G}$ have been proved. This identity is applied to find the so-called orientation-dependent chord length distribution function $F(\mathbb{D}, u, y)$, $u \in S^1$ for $\mathbb{D}$ (see also [2] – [4]):

$$b(\mathbb{D}, u) \cdot [1 - F(\mathbb{D}, u, y)] = \frac{1}{2} \int_{[\mathbb{D}]} \delta(|\chi(g)| - y)\, |\chi(g)|\, |\sin(\phi - u)|\, dg -$$

$$-\frac{1}{2} \int_{[\mathbb{D}]} \delta'(|\chi(g)| - y) \cdot |\chi(g)|^2\, |\sin(\phi - u)|\, \cot\alpha_1\, \cot\alpha_2\, dg,$$

where $b(\mathbb{D}, u)$ is the breadth function in direction $u \in S^1$, $\delta(y)$ is the Dirac's δ–function concentrated at y, α_1 and α_2 are the angles between the boundary of $\mathbb{D}$ and the line g at the endpoints of $\chi(g)$ which lie in one half-plane with respect to the inside of $\mathbb{D}$, ϕ is the direction of the line g and dg is the invariant measure in the space $\mathbf{G}$ (see [3]). $F(\mathbb{D}, u, y)$ can also be found using Matteron formula for the derivative of covariogram (see [3]). Determination of $\mathbb{D}$ by $F(\mathbb{D}, u, y)$ for all directions, is equivalent to the determination by its covariogram. Let V_n be the n-dimensional Lebesgue measure in $\mathbf{R}^n$. The function (see [5])$C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h))$, $h \in \mathbf{R}^n$ is called the covariogram of the body $\mathbf{D} \subset \mathbf{R}^n$. Here $\mathbf{D} + h = \{x + h,\ x \in \mathbf{D}\}$, and h is the vector $h = (u, y)$. Some aspects of complexity of algorithms are discussed. A practical application of these results in crystallography can be found in [5].

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Heat Transfer in Heterogeneous Materials and Partial Integro-Differential Equations

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Integro-differential equations play an important role in may physical phenomena. For instance, it appears in fields like fluid dynamics, biological models and chemical kinetics. One of the most important physical applications is the heat transfer in heterogeneous materials, where physician are looking for efficient methods to solve their modeled equations. The difficulty of solving integro-differential equations analytically made mathematician to search about efficient methods to find an approximate solution. The present article is designed to supply numerical solution of a parabolic Volterra integro-differential equation under initial and boundary conditions. We have made an attempt to develop a numerical solution via the use of Sinc-Galerkin method, the convergence analysis via the use of fixed point theory has been discussed, and showed to be of exponential order. For comparison purposes, we approximate the solution of integro-differential equation using Adomian decomposition method (ADM). Sometimes, the ADM is a highly efficient technique used to approximate analytical solution of differential equations, applicability of ADM to partial integro-differential equations has not been studied in details previously in the literature. In addition, we present numerical examples and comparisons to support the validity of these proposed methods.

Optimal uniform approximation on the angle by the harmonic functions

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In this talk we discuss the problem of the optimal uniform approximation on the angle $\Delta_\alpha = \{z \in \mathbb{C} : |\arg z| \leq \alpha/2\}$ by harmonic functions. The approximable function is a harmonic on the interior of Δ_α and satisfies some conditions on the boundary of Δ_α. The estimations of the growth of the approximating harmonic functions on $\mathbb{R}^2$ depend on the growth of the approximable function on Δ_α and it's differeentiable properties on the boundary of Δ_α.

The problem of uniform approximation on the sector by the entire functions was investigated by H. Kober [1], M.V. Keldysh [2], Mergelyan [3], N. Arakelian [4] and the other authors.The analog problem in the case for the meromorphic functions was discussed in work [5].

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Polynomial Approximation and Polynomial Inequalities in the Complex Plane

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I plan to discuss some basic questions of constructive function theory in the complex plane

Strongly Hypercyclic Operators

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We define strong hypercyclicity and investigate it for well-known hypercyclic operators. We give a criterion for strong hypercyclicty and use it in our investigation. It is shown that the existence of an invertible strongly hypercyclic operator is equivalent to giving a negative answer to the invariant subset problem.

On a general class of the finite difference schemes arising in Reaction-diffusion systems

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In this talk we shall discuss the general class of finite difference schemes developed for a numerical approximation of solutions to a certain type of reaction–diffusion systems. These schemes themselves happened to be non-linear and implicit systems. We will focus on the main issues for these schemes, i.e. the schemes solution's existence, uniqueness and convergence. We will also discuss the difference scheme for a multiphase obstacle problem as a particular case of such difference schemes.

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On some Hardy-Littlewood type inequalities for weighted spaces in reduced quaternions

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In the theory of weighted spaces, differentiation and integration operations are an important tool for studying and characterization of the spaces. Hardy and Littlewood in 1920-30s found out the precise action of differentiation and integration in weighted Bergman spaces of holomorphic functions in the unit disc of the complex plane. Hardy and Littlewood were the first who considered the problem of harmonic conjugation in Bergman spaces on the unit disc. The problems of integro-differentiation and harmonic conjugates in the framework of quaternionic and Clifford analysis were already studied by several authors.

In this talk, we prove some Hardy-Littlewood inequalities in norms for monogenic Hardy, weighted Bergman and Dirichlet spaces of reduced quaternion-valued functions in the unit ball $B_3 \subset \mathbb{R}^3$. Instead of ordinary derivatives or gradient, we apply hypercomplex derivative for monogenic functions. Also, "harmonic conjugation" operator is bounded in weighted Dirichlet spaces of quaternion-valued functions in the ball B_3.

On a Dirichlet Problem for Sixth Order Improperly Elliptic Equation

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Let $D = \{z : |z| < 1\}$ be a unit disk and $\Gamma = \partial D$ its boundary. We consider the improperly elliptic sixth order differential equation

$$\sum_{k=0}^{6} A_k \frac{\partial^6 u}{\partial x^k \partial y^{6-k}}(x,y) = 0, \quad (x,y) \in D, \tag{1}$$

where A_k are such complex constants ($A_0 \neq 0$), that the numbers λ_j ($j = 1,\ldots,6$) – the roots of characteristic equation $\sum_{k=0}^{6} A_k \lambda^{6-k} = 0$, satisfy the condition

$$\lambda_1 = \ldots = \lambda_5 \neq i,\ \Im\lambda_j > 0,\ j = 1,\ldots,5;\ \Im\lambda_6 < 0. \tag{2}$$

The solution of the equation (1) to be found in the class $C^6(D) \bigcap C^{(2,\alpha)}(\overline{D})$, and on the boundary Γ ($z = e^{i\theta}$) satisfy Dirichlet conditions:

$$\left.\frac{\partial^2 u}{\partial z^j \partial \bar{z}^{2-j}}\right|_\Gamma = F_j(\theta),\ \ u(1,0) = c_0,\ u_x(1,0) = c_1,\ u_y(1,0) = c_2. \tag{3}$$

Here $j = 0,1,2$; F_j are given functions, c_j are given constants. The case of second order improperly elliptic equation (1) was considered in [1] (see also [2]), further results and the case of fourth order equation may be found in [3]. Let $B^{(\alpha)}(r)$ be the space of the functions g, analytic in the ring $\{r < |z| < 1\}$ and Holder continuous with second order derivatives up to the boundary; and $\mu = \frac{i-\lambda_1}{i+\lambda_1}$. Then obtained result may be formulated as follows.

Theorem 1. *If the boundary functions F_j belong to the class $B^{(\alpha)}(|\mu|)$ then the problem (1), (3) has a solution and this solution is unique.*

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On the uniqueness of meromorphic functions and its derivatives sharing one set

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The subject on sharing values between entire functions and their derivatives was first studied by Rubel and Yang [2]. Subsequently, similar considerations have been made with respect to higher derivatives and more general differential expressions as well. Those results motivate the researchers to study the relation between an entire function and its derivative counterpart for one shared value counting multiplicities.

In 1996, in this direction the regarding value sharing of an entire functions together with its derivative counterpart was first studied by Bruck [2] . In course of time, there were several generalizations and extensions. Since the generalization of derivative and value is the differential polynomial and set respectively, so it will be interesting to study the relation between polynomial of a meromorphic function with its differential polynomial when they share some suitable sets. In this talk, we have investigated the uniqueness of a non-constant meromorphic function f or a polynomial of f and its linear or non-linear differential polynomial sharing a set S under different constraints. We have also tried to point out the gradual development in this context and tried to justify our certain observations by relevant examples.

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On inversion of Toeplitz matrices with elements from a ring

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Let $A = (a_{k-m})_{k,m=0}^{n}$ be an invertible Toeplitz matrix. In the case of a scalar matrix, the Baxter-Hirschman formula [3] and its modifications give an explicit form of the matrix A^{-1} through two vectors – solutions of two equations with the matrix A. In the "nonscalar" case of a matrix with elements from a ring, the matrix A^{-1} was constructed through four vectors [2]. Wherein an additional condition was imposed on the properties of these vectors, which is not necessary for the invertibility of the matrix A and considerably complicates the application of the obtained formula. The report presents a "regular" inversion formula in the nonscalar case, when additional conditions are not imposed.

Let G be the associative ring with unit e and neutral element 0, and the matrix A with elements from G has the inverse A^{-1}. Then the following four systems of equations are uniquely solvable (it is assumed that $a_{-(n+1)} = 0$):

$$\sum_{s=0}^{n} a_{m-s} x_s = \delta_{m0} e \qquad , \sum_{s=0}^{n} a_{m-s} \zeta_s = -a_{-n-1+m}\,, \quad (m = 0, 1, \cdots, n)\,,$$

$$\sum_{s=0}^{n} \eta_s a_{s-m} = \delta_{nm} e, \qquad \sum_{s=0}^{n} z_s a_{s-m} = -a_{-1-m}\,, \quad (m = 0, 1, \cdots, n)\ .$$

Theorem 1. *The following formula holds:*

$$A^{-1} = \begin{pmatrix} x_0 & 0 & 0 & \cdots & 0 \\ x_1 & x_0 & 0 & \cdots & 0 \\ x_2 & x_1 & x_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & x_{n-1} & x_{n-2} & \cdots & x_0 \end{pmatrix} \begin{pmatrix} e & z_0 & z_1 & \cdots & z_{n-1} \\ 0 & e & z_0 & \cdots & z_{n-2} \\ 0 & 0 & e & \cdots & z_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e \end{pmatrix} -$$

$$- \begin{pmatrix} \zeta_0 & 0 & 0 & \cdots & 0 \\ \zeta_1 & \zeta_0 & 0 & \cdots & 0 \\ \zeta_2 & \zeta_1 & \zeta_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \zeta_n & \zeta_{n-1} & \zeta_{n-2} & \cdots & \zeta_0 \end{pmatrix} \begin{pmatrix} 0 & \eta_0 & \eta_1 & \cdots & \eta_{n-1} \\ 0 & 0 & \eta_0 & \cdots & \eta_{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \eta_0 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}.$$

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Uniform rational approximation and optimal design of electrical filters

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The synthesis of optimal multiband electrical filters is based on solving a certain problem of uniform rational approximation reminiscent of the third problem of Zolotarev. A novel view at the formulation of problems in this area will be offered and the recent successes in solving this problem related to the use of the algebro-geometric Ansatz are described.

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Mergelyan's polynomial approximation theorem and related questions

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In 1951 Mergelyan solved the classical polynomial approximation problem in the complex plane by proving his famous theorem. Ever since the theorem became fundamental in approximation theory and complex analysis, and opened horizons of research in many directions. It is interesting to note that the powerful and constructive method of the proof created by Mergelyan in 1951 even today is the only known constructive proof of his theorem.

In this talk we discuss some of the roots of Mergelyan's theorem, its developments in different directions, as well as an open approximation problem closely related to Mergelyan's theorem.

A theorem on even pancyclic bipartite digraphs

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In this paper we consider finite digraphs without loops and multiple arcs. Terminology and notation not described below follow [4]. There are various sufficient conditions for a digraph to be hamiltonian are also sufficient for the digraph to be pancyclic (see, e.g., [4], [9]). In particular, the author [5] characterizes those digraphs which satisfy Meyniel's condition for hamiltonicity of digraphs, but are not pancyclic.

Theorem 1.1 ([5]). *Let D be a strongly connected digraph of order $n \geq 2$. Suppose that $d(x) + d(y) \geq 2n - 1$ for all pairs of distinct non-adjacent vertices x, y in D. Then D is pancyclic with some exceptions which we characterize.*

Recently, there has been renewed interest in various Meyniel-type condition for hamiltonicity and even pancylicity in bipartite digraphs (see, e.g., [1]-[3], [6]-[8], [10]). In this paper using Theorem 1.1 and some arguments of [**?**] we prove the following theorem, which improves the main result of [10].

Theorem 1.2. *Let D be a strongly connected balanced bipartite digraph of order $2a \geq 6$ with partite sets X and Y. If $d(x) + d(y) \geq 3a$ for every pair of distinct vertices $\{x, y\}$ either both in X or both in Y, then D is even pancyclic.*

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Two-Step Implicit Higher Order Numerical Integrator for Stiff Systems of Ordinary Differential Equations

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This paper is focused on the derivation of a Two-step Implicit Higher order Numerical Integrator for Stiff Systems of Ordinary Differential Equations. In this dissertation, an exponentially fitted numerical method is developed using the method of trapezoidal interpolant for the numerical integration for stiff system. This method preserves the A-stability property of numerical scheme and is also L-stable. Two theorems and one Lemma were proposed and proved which establish the A-stability and L-stability properties of the derived method. The local truncation error of the method is estimated. The derivation of the continuous form of this method is attempted and presented. The analysis show clearly that the method compete favourably with other known methods when applied to stiff system of initial value problems of ordinary differential equations. The Result obtained and the numerical error is favourably compared with those existing methods and also the theoretical solution for solving such problems.

Nevanlinna domains with large boundaries

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The concept of a Nevanlinna domain is the special analytic characteristic of bounded simply connected domains in the complex plane. It plays a crucial role in recent advances in problems of uniform approximation of functions on compact sets in the plane by polynomial solutions of elliptic equations with constant complex coefficients. In the talk we will present the final solution to the following problem posed in the early 2000s: how large (in the sense of dimension theory) there can be boundaries of Nevanlinna domains?

On the area of the numerical range

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The numerical range $W(A)$ of a Hilbert space operator A is defined by the formula $W(A) = \{\langle Ax, x\rangle : \|x\| = 1\}$. According to the classical Hausdorf-Teoplitz theorem $W(A)$ is a bounded convex subset of the complex plane $\mathbb{C}$. In [3] is proved that the self commutator $AA^* - A^*A$ of A satisfies the following inequalities

$$\|AA^* - A^*A\| \leq \inf_{N \in \mathcal{N}(A)} \|A - N\|^2$$

($\mathcal{N}(A)$ is the set of all normal operators, commuting with A) and

$$\|AA^* - A^*A\| \leq S,$$

where S is the area of the minimal rectangle, containing $W(A)$. The area of $W(A)$ may be easily calculated for operators having elliptical disk as its numerical range. For general case different estimates are known. By [4] (Theorem 30) for an $n \times n$ matrix the area S of $W(A)$ satisfies the inequality

$$\frac{1}{n}\sqrt{tr^2(AA^*) - tr(A^2)\, tr(A^{*2})} \leq S \leq$$
$$2\frac{n-1}{n}\sqrt{tr^2(AA^*) - tr(A^2)\, tr(A^{*2})},$$

where $tr(B)$ is the trace of the matrix B. For 3×3 nilpotent matrices

$$A = \begin{pmatrix} 0 & \alpha & \beta \\ 0 & 0 & \gamma \\ 0 & 0 & 0 \end{pmatrix}, \alpha, \beta, \gamma \in \mathbb{C}, |\alpha| + |\beta| + |\gamma| > 0.$$

Chien and Lin in [2] proved that the area admits the following estimate from above

$$\pi\frac{240a^2 + 624ab + 275b^2}{512},$$

where $a = \max\{|\alpha|, |\beta|, |\gamma|\}$ and b is the second largest in that set. The area of any convex compact $F \subset \mathbb{C}$ may be calculated ([1], §8, 39,(5)) by the Blaschke formula

$$S = \frac{1}{2}\int_{0}^{2\pi}\left(p^{2}(\varphi) - p'^{2}(\phi)\right)d\varphi,$$

where p is the support function of F. In this talk we find the support function for $W(A)$ and calculate the area of the numerical range for some matrices and operators acting in infinite dimensional Hilbert space and compare it with the norm of the self-commutator.

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Quantum Vacuum-Quintessence as the Natural Quantum Computer

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In the framework of complex stochastic equations type of Weyl-Langevin it is proved that the dark energy-quintessence is an ensemble of zero-spin massless particles or a quantum computer. The logic of this natural computer is analyzed in detail and shown that it is much more complicated than a register consisting of qubits which is now being implemented in practice.

On uniqueness of Franklin series

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The Franklin system, introduced by Ph. Franklin [1] in 1928, is a complete orthonormal system of continuous, piecewise linear functions with dyadic knots. It was introduced as an example of a complete orthonormal system being a basis in $C[0,1]$. Since then, it has been studied by many authors from different points of view, and various extensions and generalizations of this system has been considered.

Let $\{f_n\}_{n=0}^{\infty}$ be Franklin system on $[0,1]$. Before 2016 the problem of uniqueness of Franklin series was open.

Problem 1. *Does it follow from* $\sum_{n=0}^{\infty} a_n f_n(x) = 0$ *for each* $x \in [0;1]$ *, that* $a_n = 0$ *for all* $n = 0, 1, 2, ...$*?*

It is well-known [2], [3] that $|\sum_{n=0}^{\infty} f_n(0) f_n(t)| < CNq^N$, where C, q are some absolute constants and $0 < q < 1$. Hence, there exists a series $\sum_{n=0}^{\infty} a_n f_n(t)$, that converges to zero for $t \in (0;1]$ and $a_0 \neq 0$.

Note, that $|f_n(0)| < C\sqrt{n}$ and $\limsup_{n\to 0} \frac{|f_n(0)|}{\sqrt{n}} > 0$ (see [2], [3]).

Problem 2. *Do the conditions* $\sum_{n=0}^{\infty} a_n f_n(x) = 0$ *for* $x \in (0;1]$ *and* $a_n = o(\sqrt{n})$ *imply, that* $a_n = 0$ *for all* $n = 0, 1, 2, ...$ *?*

Problem 3. *Let E be some finite or countable set. Do the conditions*

$$\sum_{n=0}^{\infty} a_n f_n(x) = 0 \quad \text{for} \quad t \in [0;1] \setminus E, \qquad \text{and} \qquad a_n = o(\sqrt{n})$$

imply, that $a_n = 0$ *for all* $n = 0, 1, 2, ...$*?*

The answers of the above mentioned problems are positive:

Theorem 1. *If the series* $\sum_{n=0}^{\infty} a_n f_n(x)$ *converges everywhere to an everywhere finite and integrable function* $f(x)$*, then it is the Fourier-Franklin series of the function* f*. In particular, if* $f = 0$*, then all coefficients are zero.*

Theorem 2. *Let* $a_n = o(\sqrt{n})$ *and* E *be some countable set. If the series* $\sum_{n=0}^{\infty} a_n f_n(x)$ *converges everywhere, possibly except of some countable set, to an everywhere finite and integrable function* $f(x)$*, then it is the Fourier-Franklin series of the function* f*. In particular, if* $f = 0$*, then all coefficients are zero.*

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A Menger curve with the co-Hopfian property

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A mapping between metric spaces is said to be quasisymmetric (QS) if it distorts shapes in a controlled manner. A metric space X is QS co-Hopfian if every QS mapping of X into itself is onto. Motivated by geometric group theory the question of the existence of certain QS co-Hopfian spaces became of interest. In 2010 Merenkov gave the first example of a metric space homeomorphic to the Sierpinski carpet which is QS co-Hopfian and asked if there is such a space homeomorphic to the Menger curve.

We explain the construction of the first example of a QS co-Hopfian metric space that is homeomorphic to the classical Menger cure.

On a property of GC_n sets

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An n-poised node set $\mathcal{X}$ in the plane is called GC_n set if the (bivariate) fundamental polynomial of each node is a product of n linear factors. A line is called k-node line if it passes through exactly k-nodes of $\mathcal{X}$. An $(n+1)$-node line is called maximal line. The well-known conjecture of M. Gasca and J. I. Maeztu [1] states that every GC_n set has a maximal line. Until now the conjecture has been proved only for the cases $n \leq 5$ [2]. We say that a node uses a line if the line is a factor of the fundamental polynomial of this node. It is a simple fact that any maximal line M is used by all $\binom{n+1}{2}$ nodes in $\mathcal{X} \setminus M$. We consider the main result of the paper [3], stating that any n-node line of GC_n set is used either by exactly $\binom{n}{2}$ nodes or by exactly $\binom{n-1}{2}$ nodes, provided that the Gasca-Maeztu conjecture is true.

Here we show that this result is not correct in the case $n = 3$. Namely, we bring an example of a GC_3 set and a 3-node line there which is not used at all. Fortunately, then we were able to establish that this is the only possible counterexample, i.e., the above mentioned result is true for all $n \geq 1, n \neq 3$.

We also characterize the exclusive case $n = 3$ and present some new results on the maximal lines and the usage of n-node lines in GC_n sets.

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Up to Date Directions of Information Theory Research in Armenia

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Information theory is a mature field, a fundamental discipline that has great relevance in a number of vibrant research areas. The beginning of information theory in Armenia was led by R. L. Dobrushin. He repeatedly visited Armenia with lectures in Yerevan State University and Computing center of AS. Records of this lectures by PhD student E. A. Haroutunian were published in [1].

The investigations of Armenian researchers are devoted to one of the central problems of information theory the problem of determination of interdependence between coding rate and error probability exponent for various information systems. One of the contributions of E. A. Haroutunian is that he was the first who proposed different bounds of characteristics of communication systems by maxmin of functionals with entropy, information and divergence [2]. Now the Haroutunians form of presentation of different bounds is named "standard". Many of results of E. A. Haroutunian are included in the textbooks of information theory and are cited by other authors. More often the result on upper bound to the error exponent for channels with feedback is mentioned as Haroutunians exponent or Haroutunians bound [3]. Despite the numerous investigations this bound has not been improved till now.

During last years the investigations of the various problems were developed in Yerevan.

- Upper and lower bounds are constructed for E-capacity of discrete memoryless channel.
- Multiterminal channels (two-way channels, interference channels, broadcast channels, multiple-access channels) are investigated.

- Varying channels (compound channel, channel with random parameter, multiple-access channel with random parameter, arbitrarily varying channel) are studied.

- In source coding the rate-reliability-distortion function is investigated for various systems.

- The problem of logarithmically asymptotically optimal testing of statistical hypotheses in terms of error exponents is solved for various models.

- The interconnection of main characteristics of the biometric identification systems are investigated.

- Security models (information-hiding systems, generalized model of channel with side information, broadcast channels with confidential messages, wiretap channel, Shannon cipher system with the guessing wiretapper) are studied.

- The role of information theory in Community detection is expanded.

- For estimation and computations of complex formulas the new package for R environment has been developed `http://packages.reviewed.r-project-0-mirror.com/AdvInfTheo_v1.0.5.tar.gz`.

These results of authors and their students are published in more than 300 publications. A survey of a part of these results is expounded in [4].

Authors of this survey have been teaching courses of Information Theory and particularly the concept of E-capacity in Yerevan State University, in Armenian National Polytechnic University and in International Scientific Educational Center of NAS for many years and prepared teaching aids in Armenian [4, 5]. Under their guidance more than 20 PhD students defended dissertations and now are successfully working in Armenia and abroad.

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The direct and inverse Sturm-Liouville problems

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We construct the *Eigenvalue's function of the family of Sturm-Liouville operators* (EVF) and study the properties of this function.

We find, that these properties not only necessary, but also sufficient for a function of two variables be EVF of a family of Sturm-Liouville operators.

Markov–Bernstein type estimates on the Hamming cube

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Hamming cube of dimension n can be considered as the set of all vectors of length n with coordinates plus or minus 1. Functions on the Hamming cube can be expanded into the corresponding Fourier–Walsh series of degree n. Approximating such functions by simpler functions, for example, Fourier–Walsh polynomials of smaller degree (living on low frequencies) are of important interests. I will speak about Bernstein–Markov type estimates, and its converse forms, for functions on the hamming living on low frequencies, and on high frequencies correspondingly.

This is joint work with Alexandros Eskenazis.

On exceptional sets of Hilbert transform

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The Hilbert transform of a function $f \in L^1(\mathbb{R})$ is the integral

$$Hf(x) = \lim_{\varepsilon \to 0} H_\varepsilon f(x) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \int_{|t-x|>\varepsilon} \frac{f(t)}{x-t} dt.$$

It is well-known the almost everywhere existence of this limit for the integrable functions. The maximal Hilbert transform is defined by

$$H^* f(x) = \sup_{\varepsilon > 0} |H_\varepsilon f(x)|.$$

Examples of exceptional sets for the Hilbert transform have been only considered by Lusin in his famous book. It was proved the existence of an everywhere dense continuum null set $e \subset \mathbb{R}$, such that $H^* f(x) = \infty$ on e for some $f \in C(\mathbb{R}) \cap L^1(\mathbb{R})$. The following theorems shows that any null set e can serve as an exceptional set for the Hilbert transform of some indicator function. Moreover, if e is additionally compact, then instead of the indicator function it can be taken a continuous function.

Theorem 1. *For any null set $e \subset \mathbb{R}$ there exists a set $E \subset \mathbb{R}$ of finite measure such that*

$$H^* \mathbb{I}_E(x) = \infty, \quad x \in e.$$

Theorem 2. *For any closed null set $e \subset \mathbb{R}$ there exists a continuous function $f \in C(\mathbb{R}) \cap L^1(\mathbb{R})$ such that*

$$H^* f(x) = \infty, \quad x \in e.$$

The following question is open.

Problem. *Is the statement of Theorem 2 valid for arbitrary null sets.*

Growth Estimates for Weighted Classes of Holomorphic Functions in the Matrix Disc

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Let $m, n \geq 1$ be arbitrary natural numbers. Denote by M_{mn} the space of all complex $m \times n-$ matrices. For arbitrary matrix $\eta \in M_{mn}$ denote by $\eta^* \in M_{nm}$ its Hermitian conjugate matrix. Further, $I^m (m \geq 1)$ is the unit $m \times m-$matrix from M_{mm}. The Lebesgue measure in M_{mn} can be written in the following natural way:

$$d\mu_{mn}(\eta) = \prod_{k=1}^{m} \prod_{j=1}^{n} dm(\eta_{kj}), \quad \eta = (\eta_{kj})_{1 \leq k \leq m, 1 \leq j \leq n}.$$

The domain

$$R_{mn} = \{\eta \in M_{mn} : I^m - \eta \cdot \eta^* \quad is \quad positive \quad definite\} \tag{1}$$

is called a matrix unit disc (Cartan classical domain of type I). Note that this domain can be defined by the condition $||\eta|| < 1$ where $|| \bullet ||$ is the spectral (operator) matrix norm.

For $0 < p < \infty, \alpha > -1$ denote by $H^p_\alpha(R_{mn})$ the space of all holomorphic functions $f(\eta), \eta \in R_{mn}$, satisfying the condition

$$||f||_{p,\alpha} \equiv \left(\int_{R_{mn}} |f(\eta)|^p \cdot [\det(I^m - \eta \cdot \eta^*)]^\alpha d\mu_{mn}(\eta) \right)^{\frac{1}{p}} < +\infty. \tag{2}$$

Note that for $m = 1, n \geq 1 \quad R_{mn}$ is the unit ball $B_n \subset C^n$ and the space $H^p_\alpha(R_{mn}) \equiv H^p_\alpha(B_n)$ is defined by the condition

$$||f||_{p,\alpha} \equiv \left(\int_{B_n} |f(\eta)|^p \cdot (1 - |\eta|^2)^\alpha d\mu_{1n}(\eta) \right)^{\frac{1}{p}} < +\infty. \tag{3}$$

The growth of functions in $H^p_\alpha(R_{mn})$ near boundary of the domain R_{mn} can be described by the following

Theorem. *For arbitrary function* $f \in H^p_\alpha(R_{mn})(0 < p < \infty, \alpha > -1)$ *the following estimates are true:*

$$|f(z)| \leq \frac{const(m;n;p;\alpha) \cdot ||f||_{p,\alpha}}{\left[\det(I^m - z \cdot z^*)\right]^{\frac{m+n+\alpha}{p}}}, \quad \forall z \in R_{mn}; \tag{4}$$

$$|f(z)| \leq \frac{const(m;n;p;\alpha) \cdot ||f||_{p,\alpha}}{(1 - ||z||^2)^{\frac{\nu(m+n+\alpha)}{p}}}, \quad \forall z \in R_{mn}, \tag{5}$$

where $\nu = \min\{m;n\}$.

Unconditionality of periodic orthonormal spline systems in L^p

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Let $(s_n)_{n=1}^{\infty}$ be a dense sequence of points in the torus $\mathbb{T}$ such that each point occurs at most k times. Such point sequences are called *admissible*.

For $n \geq k$, we define $\hat{\mathcal{S}}_n$ to be the space of polynomial splines of order k with grid points $(s_j)_{j=1}^{n}$. For each $n \geq k+1$, the space $\hat{\mathcal{S}}_{n-1}$ has codimension 1 in $\hat{\mathcal{S}}_n$ and, therefore, there exists a function $\hat{f}_n \in \hat{\mathcal{S}}_n$ that is orthonormal to the space $\hat{\mathcal{S}}_{n-1}$. Observe that this function $\hat{f}_n$ is unique up to sign. In addition, let $(\hat{f}_n)_{n=1}^{k}$ be an orthonormal basis for $\hat{\mathcal{S}}_k$. The system of functions $(\hat{f}_n)_{n=1}^{\infty}$ is called *periodic* orthonormal spline system of order k corresponding to the sequence $(s_n)_{n=1}^{\infty}$. We remark that if a point x occurs m times in the sequence $(s_n)_{n=1}^{\infty}$ before index N, the space $\hat{\mathcal{S}}_N$ consists of splines that are in particular $(k-1-m)$ times continuously differentiable at x, where here for $k-1-m \leq -1$ we mean that no restrictions at the point x are imposed. This means that if $m = k$ and also $s_N = x$, we have $\hat{\mathcal{S}}_{N-1} = \hat{\mathcal{S}}_N$ and therefore it makes no sense to consider non-admissible point sequences.

The main result is the following

Theorem 1. *Let $k \in \mathbb{N}$ and $(s_n)_{n\geq 1}$ be an admissible sequence of knots in $\mathbb{T}$. Then the corresponding periodic orthonormal spline system of order k is an unconditional basis in $L^p(\mathbb{T})$ for every $1 < p < \infty$.*

Uniqueness theorems in non-homogeneous Carleman classes

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I will consider non-homogeneous Carleman classes consisting of smooth functions in the interval $[0,1]$. In the talk, I will describe necessary and sufficient conditions which guarantee that any function in such a class is uniquely determined by its Taylor coefficients at the origin. If time permits, I will also discuss the motivation for defining such classes.

Boundary smoothness drop for an analytic function compared to the smoothness of its modulus. A survey

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Let f be a function analytic in the unit disk and continuous up to the boundary. It is well known that, if f has no zeros inside the disk and $|f|$ satisfies the α-Hölder condition on the boundary, then the $\alpha/2$-Hölder condition in the closed disk is guaranteed for f itself, and this is best possible. For $0 < \alpha < 1$ the statement is attributed to Carleson and Jacobs (unpublished); the result was rediscovered by Havin and Shamoyan around 1970. In 1988, Shirokov extended the result to arbitrary positive Hölder exponents α.

It is remarkable that the interest to the problem has been revived since 2013, and I am going to give an overview of these recent developments. First, it has turned out that the statement can be localized: a Hölder condition for $|f|$ at only one point of the boundary implies a sort of the Hölder condition (again with the above drop of the exponent) for f at the same point. Next, some simple additional restrictions on f have been found that guarantee a smaller smoothness drop. Also, an analog of the Carleson-Jacobs-Havin-Shamoyan theorem was proved in the multidimensional case (for the complex ball). Finally, the result for the ball also has turned out to be localizable, but here a new effect, absent for the disk, emerges. These results have been obtained (partly jointly, partly individually) by A. Vasin, myself, A. Medvedev, N. Shirokov, and I. Vasiliev.

Reconstruction of the values of meromorphic functions on a compact Riemann surface via Hermite-Pade polynomials

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In the talk we consider the problem of reconstructing the values of a multivalued algebraic function from an initial germ with the help of the Hermite-Padé polynomials of the first kind.

Modeling of Coupled Heat Transport and Water Flow in Porous Media and Fractured Rock Masses

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This contribution deals with modeling of coupled heat transport and water flow in unsaturated porous media accounting for conditions of freezing and thawing. The model is based on basic conservation equations, e.g. mass conservation equation and energy conservation equation. The complete model consists of two nonlinear partial differential equations with unknown total pressure head and temperature and prescribed boundary and initial conditions. Numerical procedure is based on a semi-implicit time discretization, which leads to a system of coupled nonlinear stationary equations. The next part of this contribution deals with the existence of a weak solution to the discretized problem. We also present some illustrative numerical example compared with the practical experiment. The spacial discretization is carried out by the FE-method and it is implemented in Matlab.

Proper holomorphic maps of Reinhardt domains

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Proper holomorphic maps between two-dimensional Reinhardt domains are described. Bounded Reinhardt domains admitting proper holomorphic maps onto two-dimensional complex manifolds are classified.

Topological Data Analysis

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Persistent homology is a recent and emergent tool from the applied algebraic topology area, deeply used by the mathematicians for application to real-life situations, such as image analysis, medical imaging, and topological big data analysis in general. The key idea is to view the data as a cloud of points, associate it a chain of complexes, which produces a kind of barcodes, which encodes the persistence of data. In this talk, we will explore some works in progress to classify some written systems, to implement a shape signature algorithm, to study some behavior dysfunctional under radiation, to propose some machine learning for shape recognition. Cliques graphs will be deeply used.

From Mergelyan's theorem to universal approximation

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The celebrated theorem of Mergelyan states that, if K is a compact set in the complex plane with connected complement, then, for each continuous function g on K which is holomorphic on the interior of K, there is a sequence of polynomials which converges uniformly to g on K. It is natural to ask if all these approximating polynomials can be obtained by considering subsequences of partial sums of a single Taylor series. It turns out that the existence of such *universal* Taylor series is a generic phenomenon, and has been studied intensively over the past 20 years. In this talk, we will discuss some interesting properties of universal Taylor series and will present the important problems of the area.

A generalized characteristic for functions meromorphic in the half plane

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In the study of meromorphic functions in the complex plane, the Nevanlinna characteristics and their various generalizations are often used. However, these characteristics do not take into account the arguments of a-points of functions. Therefore, in the theory of the distribution of values of meromorphic functions, other characteristics that take into account the angular distributions of the a-points are used (see for example [1]). In [2], for functions meromorphic in the half-plane, the authors attribute the Tsuji characteristics generated by Levin's formula to such characteristics.

We generalize Levin's formula [2], paraphrasing the results for the lower half-plane as in [3], after which Levin's formula and the Tsuji characteristics acquire more natural form. Then, applying the method of Fourier transforms for meromorphic functions (see [4]), we consider Levin's formula as the value of the Fourier transform at the point $x = 0$ and introduce generalized characteristics for functions meromorphic in the lower half-plane and generalize the first fundamental theorem of Tsuji.

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Rigidity, Graphs and Hausdorff Dimension

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We prove that if $E \subset \mathbb{R}^d$ is any compact set of Hausdorff dimension larger than $s_d(k) = d - \frac{1}{k+1}$, then the m-dimensional Lebesgue measure of the set of congruence classes of $(k+1)$-point configurations of points from E is positive.

This can be viewed as a generalization of the Falconer distance problem ([1]) on one hand, and of the Furstenberg-Katznelson-Weiss (see e.g. [2], [3]) type configuration results on the other. The proof relies on analytic, combinatorial and topological considerations.

This is joint work with Nikolaos Chatzikonstantinou, Alex Iosevich and Jonathan Pakianathan.

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Estimates for Strong-Sparse Operators

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Let $\mathcal{S}$ be a sparse collection of dyadic intervals in R^d. Our interest is in weighted L^2 bound of the operator

$$S^*f = \sum_{B\in\mathcal{S}} \chi_B \cdot \sup_{A\supset B} \frac{1}{|A|}\int_A f.$$

It is trivial, that $S^*f \leq S(Mf)$ which gives $\|S^*\|_{L^2(w)\to L^2(w)} \leq [w]_{A_2}^2$. We prove, the sharp bound $\|S^*\|_{L^2(w)\to L^2(w)} \leq [w]_{A_2}^{3/2}$. The techniques are those of stopping cubes, Sawyer-type testing conditions and corona decomposition, in particular a localization method introduced by Lacey-Sawyer and Uriarte-Tuero.

Universal Taylor Series without Baire's method and two theorems of Mergelyan

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I will present my first proof of the existence of universal Taylor series on the disc, which was done in 1995. It was a construction without the use of Baire's method and it was based on Mergelyan's famous theorem. A less known result of Mergelyan was part of his thesis and states that there exists a domain G in the plane supporting a function f in $A(G)$, whose primitive is unbounded on G. In collaboration with Ilias Zadik we gave a different proof of the above fact, which appeared in JMAA (2015).

Averaged fractional controllability

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In the paper, we generalize results concerning averaged controllability on fractional type equations: system of fractional ODE-s and the fractional diffusion equation.

Fourier approach to mean-field game systems

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Mean-field games (MFG) is a recently introduced framework to study huge populations of identical agents that play non-cooperative differential games. More precisely, given a very large population of agents that play a non-cooperative differential game, we aim at describing Nash equilibria of this game. Since the number of players is very large, the equilibria conditions yield a huge system of partial differential equations (PDE) that is not tractable from mathematical and computational perspectives. Thus, reminiscent of the statistical-physics approach, for each agent one models actions of the others in an aggregated sense. This simplification yields a model where a typical agent plays against the population as a whole – the *mean-field*. Hence, instead of finding all individual strategies, one searches for optimal actions of a typical player and the distribution of the population when everyone acts optimally.

Mathematically, an MFG model is a nonlinear system of PDE that consists of a Hamilton-Jacobi-Bellman (HJB) PDE coupled with a Kolmogorov-Fokker-Planck (KFP) PDE. Former determines optimal actions of a typical agent whereas latter describes the distribution of the population when agents act optimally. In this talk, we discuss several new ideas towards using Fourier analysis techniques to study MFG systems.

Covariogram of convex bodies and geometric probabilities

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Let $\mathbf{R}^n$ $(n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body with inner points, and V_n be the n-dimensional Lebesgue measure in R^n. The function $C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in \mathbf{R}^n$ is called the covariogram of the body $\mathbf{D}$. Here $\mathbf{D} + h = \{x + h,\ x \in \mathbf{D}\}$ (see [1]). We consider a random line which is parallel to $\mathbf{u} \in S^{n-1}$ and intersects $\mathbf{D}$, that is, an element from the set:

$$\Omega_1(\mathbf{u}) = \{\text{lines which are parallel to } \mathbf{u} \text{ and intersect } \mathbf{D}\}.$$

Let $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ be the orthogonal projection of $\mathbf{D}$ onto the hyperplane $\mathbf{u}^\perp$ (here $\mathbf{u}^\perp$ stands for the hyperplane with normal $\mathbf{u}$, passing through the origin). A random line which is parallel to $\mathbf{u}$ and intersects $\mathbf{D}$ has an intersection point (denoted by x) with $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$. We can identify the points of $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ and the lines which intersect $\mathbf{D}$ and are parallel to $\mathbf{u}$. Assuming that the intersection point x is uniformly distributed over the convex body $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$, we can define the following distribution function. The function $F(\mathbf{u}, t) = \frac{V_{n-1}\{x \in \Pi r_{\mathbf{u}^\perp}\mathbf{D} : V_1(g(\mathbf{u}, x) \cap \mathbf{D}) < t)\}}{b_{\mathbf{D}}(\mathbf{u})}$ is called orientation-dependent chord length distribution function of $\mathbf{D}$ in direction $\mathbf{u}$ at point $t \in R^1$, where $g(\mathbf{u}, x)$ is the line which is parallel to $\mathbf{u}$ and intersects $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ at point x and $b_{\mathbf{D}}(\mathbf{u}) = V_{n-1}(\Pi r_{\mathbf{u}^\perp}\mathbf{D})$ (see [2]). Denote by $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ probability, that random segment $L(\mathbf{u}, \omega)$ (of fixed length l and direction $\mathbf{u}$) entirely lying in body $\mathbf{D}$. Probability $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ in terms of distribution function $F(\mathbf{u}, z)$ has the following form (see [3]): $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{V_n(\mathbf{D}) - l\, b_{\mathbf{D}}(\mathbf{u}) + b_{\mathbf{D}}(\mathbf{u}))\int_0^l F(\mathbf{u}, z)\, dz}{V_n(\mathbf{D}) + l\, b_{\mathbf{D}}(\mathbf{u})}$, while in the terms of the covarigram of body $\mathbf{D}$ has the form: $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{C(\mathbf{D}, \mathbf{u}, l)}{V_n(\mathbf{D}) + l\, b_{\mathbf{D}}(\mathbf{u})}$,

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The calculations of mesio-distal and vestibule-lingual data by regression analysis

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Mesio-distal and vestibular-lingual diameters (i.e. maximal sizes in these dimensions) of 10 teeth crowns were measured manually and by digital automated method. Some of parameters were chosen for re-measuring to verify the accuracy of the obtained results. As it has been the first experience of application of two above-mentioned methods to measurements of anthropological samples, we were interested in comparing of the estimated parameters of tooth crowns. The manual and digital results were compared with each other in two separate series mesio-distal (MD) and vestibular-lingual (VL). The obtained results were calculated by the linear regression method and were found the equations (1), (2). Were found the mean approximation errors (3), (4):

$$\widehat{y} = 0.939x + 0.7299 \qquad (1); \ \widehat{y} = 1.0079x - 0.204 \qquad (2)$$

(1) is the equation of the linear regression for VL, (2) is the equation of the linear regression for MD.

$$\overline{A}_1 \approx 2.5892\% \qquad (3); \ \ \overline{A}_1 \approx 0.1725 \quad (4)$$

(3) is the mean approximation for VL;(4) is the mean approximation for MD.

This investigation has showed certain differences between results obtained by application of manual measurement methods widely accepted in anthropology and automated digital odontometry. In view of the fact that the same teeth were measured though two methods it would have been difficult to expect obtaining widely divergent parameters. This can be seen on the results of calculation of mean approximation error, thus both methods can be applied for odontometry. The same calculations reveal differences in MD and VL measurements. Mesio-distal (MD) direction of measurements provides more uniform results in comparison to vestibulo-lingual (VL).

Image Caption Generation based on Object Detector

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Automated semantic information extraction from the image is a difficult task. There are works which can extract image caption or object names and their coordinates. This work presents a merged single model of object detection and automated caption generation systems. The final model extracts from image caption and object coordinates with their names without losing accuracy according to initial models.

Constructive Cognitive Models for Competition, Defense and Dialog Problems

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1. Cognitive systems represent realities, particularly, our utilities, have varying effectiveness with respect to our goals and are processed to support utilization and gaining the benefits from the utilities.

Classifying cognitive systems are effective with respect to the goals insofar as they provide appropriate utilities regularly, i.e. are modeling the utilities constructively and adequately.

We specify ontological, constructive and systemic models of cognitive systems comparable by expressiveness with algorithms and natural languages, provide arguments of their adequacy for explaining, understanding and human-computer interactions as well as convince to follow the ideas of inventors of algorithms in adequate modeling of mental behavior [1].

2. To prove the adequacy of cognitive models we consider a class of combinatorial problems defined as games where spaces of solutions are Reproducible Game Trees (RGT) [2-6].

RGT class includes important problems like computer networks intrusion protection, optimal management and marketing strategy elaboration in competitive environments, defense of military units from a variety types of attacks, communication problems, certain types of teaching.

We study, particularly, models of

- advanced RGT expert knowledge presentation and providing an experimental evidence of its adequacy to one of experts.
- RGT strategy search able to regularly acquire both common and personalized expert knowledge and to its effective usage in RGT solving.

- RGT knowledge processing for interactive personalized tutoring.

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Fourier frames and spectral problems

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I will discuss a connection between sampling measures in Paley-Wiener spaces (Fourier frames) and inverse spectral problems for differential operators. The connection is established via the use of truncated Toeplitz operators and Krein-de Branges theory of Hilbert spaces of entire functions. At the end of the talk I will present several new examples of solutions of inverse spectral problems for canonical Hamiltonian systems. The talk is based on joint work with N. Makarov.

Nonlocal Contact Problems for Solution of Some Linear Equation of Mathematical Physics

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Nonlocal boundary and initial- boundary problems represent very interesting generalizations of classical problems. At the same time, they quite often arise during the creation of mathematical models of real processes and the phenomena in physics, engineering, ecology, etc. The history of investigation of nonlocal problems begins in the first half of the last century and now they are developing rapidly due to their great practical and theoretical value. In the present report, the boundary and initial-boundary problems with nonlocal contact conditions are investigated for the linear partial differential equations of elliptic and parabolic types with variable coefficients. Existence and uniqueness of regular solution is proved. The iterative procedure is constructed, by means of which the solution of an initial problem is reduced to the solution of sequence of classical Dirichlet problems (for the elliptic equations) and Cauchy-Dirichlet problems (for the parabolic equations). The parallel algorithms for the solution of these problems are considered. Numerical results of the solution of some specific problems for the elliptic and parabolic equations are given. In the second part of the report, a method of separation of variables (also known as the Fourier method) for some stationary and non-stationary problems with nonlocal contact conditions is considered.

Universal Teichmueller space: non-trivial example of infinite-dimensional complex manifolds

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At the moment we cannot say that there is a well-developed theory of infinite-dimensional complex manifolds. So it is important to have different examples of such manifolds. One of such examples is provided by the universal Teichmüller space and we shall present in our talk main complex geometric features of this remarkable infinite-dimensional manifold. The universal Teichmüller space $\mathcal{T}$ is the space of normalized quasisymmetric homeomorphisms of the unit circle S^1, i.e. orientation-preserving homeomorphisms of S^1, extending to quasiconformal maps of the unit disk Δ and fixing three points on S^1. It is a complex Banach manifold with the complex structure provided from Bers embedding of $\mathcal{T}$ into the complex Banach space of holomorphic quadratic differentials in a disk. The name of $\mathcal{T}$ is motivated by the fact that all classical Teichmüller spaces $T(G)$, associated with compact Riemann surfaces, are contained in $\mathcal{T}$ as complex subspaces. Another important subspace of $\mathcal{T}$ is given by the space $\mathcal{S}$ of normalized orientation-preserving diffeomorphisms of S^1. The space $\mathcal{S}$ is a Kähler Frechet manifold provided with a symplectic structure compatible with the complex structure of $\mathcal{S}$. We construct a Grassmann realization of $\mathcal{T}$ by embedding it into the Grassmann manifold of a Hilbert space which coincides with the Sobolev space $V = H_0^{1/2}(S^1, \mathbb{R})$ of half-differentiable functions on the circle. This embedding realizes the group $\mathrm{QS}(S^1)$ of quasisymmetric homeomorphisms of S^1 as a subgroup of symplectic group $\mathrm{Sp}(V)$. It also defines an embedding of $\mathcal{T}$ into the space of complex structures on V compatible with symplectic structure. The latter space may be considered as an infinite-dimensional Siegel disk.

Free boundaries on Lattice, and their scaling limits

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Probably the most well-known fact in classical potential theory is the mean value property for harmonic functions over spherical shells or balls. We shall discuss similar properties for harmonic functions on the lattice $s\mathbb{Z}^2$, and show (through numerics) that interesting new objects may appear when the size of lattice s tends to zero.

These objects have been studied in two recent works in collaboration with Hayk Aleksanyan.

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Approximation of vector fields by harmonic gradients

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Limiting Theorems for Higher-Order Differences of Random Independent Events

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In papers [1-4] the higher order differences of deterministic and random time series are studied. Thus, in [1] the higher-order difference version of the Lyapunov exponent is suggested and numerically studied, in [2] and [3, 4] the bi-stability of higher-order differences of periodic signals and the presence of full randomness in the higher-difference structure of two-state Markov chains, are established.

In this report we formulate some theorems on limiting behavior of the differences (when their order converges to infinity), taken from progressive terms of the given series of random independent events. The theorems proved are formulated in terms of some set-measures, conventionally called [3, 4] discrete capacities. The formulations of the limiting theorems involve some exceptional sets, which satisfy the Wiener criterion type relations.

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A remark about approximation by means of trigonometric polynomials

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Let E be a union of finite number of segments of real axis, $E = \bigcup_{k=1}^{n} [a_k, b_k]$, $n \geq 1$; if $n > 1$ then we assume that those segments are pairwise disjoint. We suppose that a following property is valid:

$$E \cap (E + 2\pi\nu) = \varnothing,\ \nu \in \mathbb{Z},\ \nu \neq 0. \qquad (1)$$

Let $\omega(x)$ be a modulus of continuity satisfying a condition

$$\int_0^x \frac{\omega(t)}{t} dt + x \int_x^\infty \frac{\omega(t)}{t^2} dt \leq c\ \omega(x) \qquad (2)$$

We denote through $H^\omega(E)$ a standard Hölder class of functions on E and let $H^{r+\omega}(E)$ be a class of functions f on E such that $f^{(r)} \in H^\omega(E)$, $r \geq 1$. Our further notations follow:

$$S_\rho = \left\{ z \in \mathbb{C} : z = \frac{1}{2}\left(\xi + \frac{1}{\xi}\right),\ |\xi| = \rho \right\},\ \rho > 1;$$

$$S_\rho([a, b]) = \frac{a+b}{2} + \frac{b-a}{2} S_\rho,\ a < b;$$

$$d_\rho(z; [a, b]) = dist\left(z, S_\rho([a, b])\right),\ z \in \mathbb{C}.$$

If $x \in [a_m, b_m] \subset E$ then $d_\rho(x) = d_\rho(x, [a_m, b_m])$.

Theorem a) *Let a set E satisfy a condition (1) and a modulus of continuity ω satisfy a condition (2). Then for any function $f \in H^{r+\omega}(E)$ and for $k \in \mathbb{N}$ there exist trigonometric polynomials $\pi_k(f, x)$ of order $\leq k$ such that a following estimate holds:*

$$|f(x) - \pi_k(f, x)| \leq c_f\ d^r_{1+\frac{1}{k}}(x)\ \omega(d_{1+\frac{1}{k}}(x)),\ x \in E. \qquad (3)$$

b) *Assume that for a function* f_0 *for any* $k \in \mathbb{N}$ *there exists a trigonometric polynomial* π_k *such that the estimate (3) is true with a constant* C_0*. Then* $f_0 \in H^{r+\omega}(E)$.

Spectral measures of finitely valued stationary sequences and all that jazz

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We will discuss a somewhat striking spectral property of finitely valued stationary sequences that says that if the spectral measure of the process has a gap then the process is periodic. We will give some extensions of this result, discuss a closely connected challenging problem pertaining to orthogonal polynomials on the unit circle, and raise several related questions. The talk is based on joint works with A. Borichev, A. Nishry, and B. Weiss (arXiv:1409.2736, arXiv:1701.03407) and on a work in progress with A. Borichev and A. Kononova.

On the quasi-greedy constant of the Haar subsystems in $L^1(0,1)$

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The better estimate for quasi-greedy constant for Haar subsystems in $L^1(0,1)$ is obtained. The known exponential dependence is improved by linear estimation.

It is known that the Haar system is not a quasi-greedy basis in $L^1(0,1)$, however there are Haar subsystems that are quasi-greedy bases in the closure of their linear span (see [1]). The complete characterization of all quasi-greedy subsystems of the Haar system is given in [2]. In that paper author obtained the following estimation for quasi-greedy constant

$$\|G_n\| \leq 2^H \tag{1}$$

where H is the length of the maximal chain of the subsystem. We improved that estimation and show that

$$\frac{H}{8} \leq \|G_n\| \leq 2H \tag{2}$$

More detailed on greed algorithms and quasi-greedy bases one can read in [3] and [4].

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On extremal property of Delaunay triangulation

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The popular method for numerical solution of some problems of mathematical physics is the finite elements method. This method needs a mesh of triangles. The convergence rate of process of numerical solution of the problem by the finite elements method depends on geometrical configuration of the mesh.

We prove the following extremal property: *The sum of cotangents of interior angles as a function on meshes with fixed set of knots reaches his minimum for Delaunay triangulation*.

Using this extremal property, the theorem is obtained, that for any fixed knots set, for numerical solution of Maxwell equation of magnetic field the optimal mesh is Delaunay triangulation.

Non-homogeneous harmonic analysis, Geometric Measure Theory and fine structures of harmonic measure

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One of the goals of harmonies analysis is to study singular integrals. Singular integrals are ubiquitous objects in PDE and in Mathematical Physics, and as it turned out recently, play an important part in Geometric Measure Theory. They have various degrees of singularity, and the simplest ones are called CalderonZygmund operators. Their theory was completed in the 50?s by Zygmund and Calderon. Or it seemed like that. The last 20 years saw the need to consider CZ operators in

very bad environment, so kernels are still very good, but the ambient set has no regularity whatsoever.

Initially such situations appeared from the wish to solve some outstanding problems in complex analysis: such as Painleves, Ahlfors, Denjoys and Vitushkins problems.

But recently it turned out that the non-homogeneous harmonic analysis (=the analysis of CZ operators on very bad sets and measures) is also very fruitful in the part of Geometric Measure Theory that deals with rectifiability, and also helps a lot to understand the geometry of harmonic measure. Lennart Carleson, Nikolai Makarov, Jean Bourgain, Peter Jones and Tom Wolff obtained important results on metric properties of harmonic measure in the 80s and 90s. But most of the results concerned the structure of harmonic measure of planar domains. As an example of the use of non-homogeneous harmonic analysis, we will show how it allows us to understand very fine property of harmonic measure of any domain in any dimension.

Distribution of poles of optimal rational functions

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Ever since the work of Runge in the late 19th century, it is known that functions analytic in a neighborhood of a compact set can be approximated arbitrarily close by rational functions (later Vitushkin characterized the compacta on which such an approximation is possible). Early in the 20th century, Walsh has shown that

$$\limsup_{n\to\infty} \inf_{r\in\mathcal{R}_n} \|f-r\|_K \le \inf_F \exp\left\{-1/\operatorname{cap}(F,K)\right\},$$

where f is holomorphic in a neighborhood of a continuum K, $\mathcal{R}_n$ is the set of rational functions of type (n,n), $\operatorname{cap}(F,K)$ is the condenser capacity, and the infimum on the right is taken over all compact sets F such that f is holomorphic in the complement of F (the complement must be connected and necessarily contain K). In general this bound is sharp. Driven by evidence from certain classes of functions, Gonchar has conjectured that

$$\liminf_{n\to\infty} \inf_{r\in\mathcal{R}_n} \|f-r\|_K \le \inf_F \exp\left\{-2/\operatorname{cap}(F,K)\right\}.$$

This conjecture was shown to be true by Parfenov with the help of Adamyan-Arov-Krein approximants. Elaborating on the work of Stahl, Gonchar and Rakhmanov have shown that

$$\lim_{n\to\infty} \inf_{r\in\mathcal{R}_n} \|f-r\|_K = \inf_F \exp\left\{-2/\operatorname{cap}(F,K)\right\}$$

if f is a multi-valued function meromorphic outside of a compact polar set. For a subclass of such functions, asymptotic distribution of poles of sequences of rational approximants $\{r_n\}$ such that

$$\lim_{n\to\infty} \|f-r_n\|_K = \inf_F \exp\left\{-2/\operatorname{cap}(F,K)\right\},$$

where K is a continuum, will be discussed. This is joint work with L. Baratchart and H. Stahl.

Method of semi-inverse factorization

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Let G be the additive semigroup of the ring B with unit I and $G = G^+ \oplus G^-$, where $G^\pm$ are subrings of B. Let $P^\pm$ are projectors, mapping G into $G^\pm$.

It is assumed that: $I \notin G^+$; if $X^+ \in G^+$, then $I - X^+$ is left invertible; if $X^\pm \in G^\pm$, then the product $X^- X^+ \in G$.

Elements $I - X^\pm$, where $X^\pm \in G^\pm$, we call normally invertible if

$$\exists \left(I - X^\pm\right)^{-1} = I + Y^\pm, \quad Y^\pm \in G^\pm.$$

Let $A \in G$. The following factorization problems can be considered:

a) Direct factorization:

$$I - A = \left(I - X^-\right)\left(I - X^+\right), \quad X^\pm \in G^\pm. \tag{1}$$

b) Inverse factorization:

$$(I - A)^{-1} = \left(I + Y^+\right)\left(I + Y^-\right), \quad Y^\pm \in G^\pm. \tag{2}$$

Factorization (1) we will call canonical if the factors $I - X^\pm$ are normally invertible. Then from (1) follows the existence of $(I - A)^{-1}$ and (2).

Suggested method of semi-inverse factorization (SIF) is based on joint consideration of the following two auxiliary problems of SIF (or one of them). Let $\in G$. It is required to find $X^\pm \in G^\pm$, such that

$$(I - A)\left(I + Y^+\right) = I - X^-, \ \left(I + Y^-\right)(I - A) = I - X^+, \quad \text{where} \quad X^\pm \in \Omega_\pm. \tag{3}$$

Consideration the first of problems (3) leads to the following equation for Y^+:

$$Y^+ = P^+ (A) + P^+ \left(AY^+ \right).$$

For X^- we obtain the expression: $X^- = P^- (AY^+)$.

To similar relations for Y^- and X^+ is reduced the second of the problems (3).

The study of the marked relationships between $X^\pm$, $Y^\pm$ led to new results (see for example in [1]), as well as to a simple derivation of a number of classical results on the existence and construction of the canonical (and regular) factorization of integral operators, matrices, etc., including the problem of linear algebra on the decomposition of a matrix into the product of two triangular matrices and the derivation of Gelfand-Levitan type equations.

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