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Abstracts

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Characterization of hyperidentities defined by the equality $((x,y),u,v)=(x,(y,u),v)$

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The following universal formula from a second-order language with specialized quantifiers have been studied in various domains of algebra and its applications and they were called hyperidentity:

$$\forall X_1,\ldots,X_m \forall x_1,\ldots,x_n (W_1 = W_2), \tag{1}$$

where w_1, w_2 are terms (words) in the functional variables $X_1,...,X_m$ and in the object variables $x_1,...,x_n$. For simplicity the hyperidentity is written without a quantifier prefix, i.e. as an equality: $w_1 = w_2$. The number m is called functional rank and the number n is called object rank of the given hyperidentity. A hyperidentity is true (or satisfied) in an algebra $(Q;U)$ if the equality $w_1 = w_2$ is valid when every object variable and every functional variable in it is respectively replaced by any arbitrary element of Q and any operation of the corresponding arity from U (it is assumed that such replacement is possible).

An algebra with binary and ternary operations is called {2,3}-algebra. A {2,3}-algebra $(Q;U)$ is called:

a) functionally non-trivial if the sets of its binary and ternary operations are non-singleton;

b) $2q$-algebra if there exists a binary quasigroup operation in U;

c) $3q$-algebra if there exists a ternary quasigroup operation in U;

d) invertible algebra if its every operation is a quasigroup operation.

Theorem 1. *If in the functionally non-trivial invertible {2,3}-algebra the hyperidentity, which is defined by the equality:*

$$((x,y),u,v) = (x,(y,u),v),$$

is satisfied, then every functional variable is repeated in it, at least, twice. Therefore, such hyperidentity can only be of functional rank 2 and it has the following form:

$$Y(X(x,y),u,v)) = Y(x,X(y,u),v).$$

Theorem 2. *If in the functionally non-trivial invertible* $2q$*-algebra the hyperidentity, which is defined by the equality:*

$$((x, y), u, v) = (x, (y, u), v),$$

is satisfied then every ternary functional variable is repeated in it, at least, twice. Therefore, such hyperidentity can only be of functional rank 2 or 3 and it has one of the following forms:

$$Y(X(x, y), u, v)) = Z(x, X(y, u), v),$$

$$Y(X(x, y), u, v)) = Y(x, X(y, u), v).$$

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Orientation-dependent events in geometric probabilities

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Let $\mathbf{R}^n$ $(n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body with inner points, and V_n be the n-dimensional Lebesgue measure in $\mathbf{R}^n$. The function

$$C(\mathbf{D}, h) = V_n(\mathbf{D} \cap (\mathbf{D} + h)), \quad h \in \mathbf{R}^n,$$

is called the covariogram of $\mathbf{D}$. Here $\mathbf{D} + h = \{x + h, \, x \in \mathbf{D}\}$.
Let S^{n-1} denote the $(n-1)$-dimensional sphere of radius 1 centered at the origin in $\mathbf{R}^n$ and $\Pi r_{\mathbf{u}^\perp}\mathbf{D}$ be the orthogonal projection of $\mathbf{D}$ onto the hyperplane $\mathbf{u}^\perp$ (here $\mathbf{u}^\perp$ stands for the hyperplane with normal $\mathbf{u}$, passing through the origin). Let $L(\mathbf{u}, \omega)$ be a random segment of length $l > 0$, which is parallel to a given fixed direction $\mathbf{u} \in S^{n-1}$ and intersects $\mathbf{D}$. Denote by $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ probability, that random segment $L(\mathbf{u}, \omega)$ (of fixed length l and direction $\mathbf{u}$) entirely lying in body $\mathbf{D}$.

Proposition 1. *Probability* $\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D})$ *in terms of covariogram of body* $\mathbf{D}$ *has the form:*

$$\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{D}) = \frac{C(\mathbf{D}, \mathbf{u}, l)}{V_n(\mathbf{D}) + l \, b_{\mathbf{D}}(\mathbf{u})},$$

where $b_{\mathbf{D}}(\mathbf{u}) = V_{n-1}(\Pi r_{\mathbf{u}^\perp}\mathbf{D})$. In particular, for the case of n-dimensional ball we obtain

$$\mathbf{P}(L(\mathbf{u}, \omega) \subset \mathbf{B_n}(\mathbf{R})) = \frac{2R}{\left(R \frac{\sqrt{\pi}\Gamma((n+1)/2)}{\Gamma(n/2+1)} + l\right)} \int_0^{\phi} \sin^n \theta \, d\theta.$$

Obviously, we have $\mathbf{P}(L(\omega) \subset \mathbf{B_n}(\mathbf{R})) = 1$ for $l = 0$ and $\mathbf{P}(L(\omega) \subset \mathbf{B_n}(\mathbf{R})) = 0$ for $l \geq 2R$.

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An approach to the spherical mean Radon transform with detectors on a line

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Introduction. Medical tomography has had a huge impact on medical diagnostics. The classical Radon transform maps a function to its integrals over straight lines and serves as the basis of x-ray Computer Tomography. Recently researchers have been developing novel methods that combine different physical types of signals. The most successful example of such a combination is the thermoacoustic tomography (TAT). Thermoacoustic theory has been discussed in many literature reviews such as [2].

Section 1. We denote by $\mathbf{R}^3$ the Euclidean 3 - dimensional space. Let $\mathbf{S}^1$ be the unit circle with the center at the origin $O \in \mathbf{R}^2$. By $S(p,r)$ we denote the circle of radius $r>0$ centered at $p \in \mathbf{R}^2$.

We consider the circular Radon transform on the plane. For a continuous function f supported in the compact region $G \in \mathbf{R}^2$ we have

$$Mf(p,r) = \frac{1}{2\pi}\int_{\mathbf{S}^1} f(p+r\omega)\,d\varphi, \quad for \;\; (p,r) \in L \times [0,\infty). \tag{1}$$

Here $d\varphi$ is the circular Lebesgue measure on $\mathbf{S}^1$, $\omega = (\sin\varphi, \cos\varphi)$. The value $Mf(p,r)$ is the average of f over the circle $S(p,r)$ with center $p \in L$ and radius $r>0$

TAT motivated the study of the following mathematical problem. For a continuous, real valued function f supported in a compact region G, we are interested in recovering f from the mean value $Mf(p,r)$ of f over spheres $S(p,r)$ centered on L; that is, given $Mf(p,r)$ for all $p \in L$ and $r>0$, we wish to recover f.

In order to implement the TAT reconstruction the following problems arise. For which sets L the data collected by transducers placed along L is sufficient for unique reconstruction of f and what are inversion formulas.Agranovsky and Quinto in [1] have proved several significant uniqueness results for the spherical Radon transform.

Exact inversion formulas for the spherical Radon transform are currently known for boundaries of special domains, including spheres, cylinders and hyperplanes ([2]).

The article suggests a new approach what is called a consistency method for the inversion of the spherical Radon transform in 2D with detectors on a line. By means of the method a new iteration formula was found which give an practical algorithm to recover an unknown function supported in a compact region from its spherical means over circles centered on a line outstand the region.

The consistency method, suggested by the author of the paper, first was applied in [3] to inverse generalize Radon transform on the sphere.

In this paper was proved the following theorem. On the plane consider usual cartesian system of coordinate choosing L as the x-axis. $Mf(p, r)$ is the average of f over a circle with center $p \in L$ and radius $r > 0$.

Theorem 1. *Let f be a continuous, real valued function supported in the compact region G located on one side of the line L. For $(x, y) \in G$ the value $f(x, y)$ depends on values Mf on a neighborhood of $p = (x, 0) \in L$ and* $0 \leq r \leq y$.

Also was found an iteration inversion formula.

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On harmonic conjugation problem in spaces of quaternion-valued functions

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The classical M. Riesz theorem on harmonic conjugates in the Hardy spaces over the unit disc $\mathbb{D}$ asserts that for a harmonic function u in the Hardy space $h^p(\mathbb{D})$ for some $p, 1 < p < \infty$, its harmonic conjugate v is also in $h^p(\mathbb{D})$, and so the holomorphic function $f = u + iv$ is in $H^p(\mathbb{D})$. Hardy and Littlewood revealed the same harmonic conjugation property for Bergman spaces in $\mathbb{D}$. Later, analogous results were obtained for various function spaces such as weighted Bergman, Bloch, Dirichlet and others. The problem of harmonic conjugates in the framework of quaternion analysis was studied by Sudbery [1] who established an explicit formula for harmonic conjugates in $\mathbb{R}^4$ such that a quaternion-valued monogenic function is defined. By another integral formula we construct harmonic conjugates in reduced quaternions in $\mathbb{R}^3$ and prove the preservation of weighted Bergman and Dirichlet spaces under harmonic conjugation operator over the unit ball in $\mathbb{R}^3$. Some preceding results can be found in [2], [3].

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On Defect Numbers of the Dirichlet Problem

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Introduction. We consider the Dirichlet problem for a sixth order properly elliptic equation in a unit disc. The new formula for the determination of the defect numbers of the problem is obtained.

Formulation of the problem and obtained result. Let D be the unit disk of the complex plane and $\Gamma = \partial D$ its boundary. We consider in D sixth order differential equation

$$\sum_{k=0}^{6} A_k \frac{\partial^6 u}{\partial x^k \partial y^{6-k}}(x,y) = 0, \quad (x,y) \in D, \tag{1}$$

where A_k are complex constants ($A_0 \neq 0$). We suppose that the roots λ_j ($j = 1, \ldots, 6$) of characteristic equation $\sum_{k=0}^{6} A_k \lambda^{6-k} = 0$, satisfy the condition

$$\lambda_1 = i \neq \lambda_2 = \lambda_3, \ \lambda_4 = \lambda_5 = \lambda_6 \neq -i, \ \Im\lambda_3 > 0, \quad \Im\lambda_4 < 0. \tag{2}$$

We consider the Dirichlet problem in the classical formulation, that is, we seek the solution of (1) in the class $C^6(D) \bigcap C^{(2,\alpha)}(\overline{D})$ satisfying Dirichlet conditions on the boundary Γ.

$$\left.\frac{\partial^j u}{\partial r^j}\right|_{\Gamma} = f_j(x,y) \quad j = 0,1,2; (x,y) \in \Gamma. \tag{3}$$

Here $f_j \in C^{(2-j,\alpha)}(\Gamma)$, ($j = 0,1,2$) are given functions. The condition (2) implies that the equation (1) is properly elliptic, therefore the problem (1), (3) is Fredholmian (see [1], [2]). The main goal of the talk is the determination of the defect numbers of this problem (the number of linearly independent solutions of the homogeneous problem (when $f_j \equiv 0$) and the number of linearly independent conditions necessary and sufficient for the solvability of the inhomogeneous problem). The general formula for the calculation of the defect numbers was obtained in [3], but this formula only provides the algorithm for the determination of these numbers. Further, in [4], it was found that for some cases of fourth order equation (1) the defect numbers may only be zero or one. In this talk we get the new formula for the determination of the defect numbers of the problem (1), (3).

The following statement is proven.

Theorem 1. *Let's denote*

$$\mu = \frac{i - \lambda_3}{i + \lambda_3}, \quad \nu = \frac{i + \lambda_4}{i - \lambda_4}, \quad z = \mu\nu.$$

Then the Dirichlet problem (1), (3) is uniquely solvable if and only if

$$Q_n(z) \equiv \sum_{k=0}^{n-3}(k+1)(k+2)(k+3)(k+4)(k+5)z^k +$$

$$+\sum_{k=0}^{n-3}(n-k-3)(n-k-2)(n-k-1)(n^2+11n+$$

$$+8nk+k^2-k)z^{n-2+k} \neq 0, \quad n = 4, 5, \ldots,$$

If these conditions fail for any number n_0, then the homogeneous problem (1), (3) has nontrivial solution, which is polynomial of order n_0+2. In this case one linearly independent condition on the boundary functions f_j is necessary for the solvability of the corresponding inhomogeneous problem. Therefore, the defect numbers of the problem (1), (3) are equal to the number of parameters n for which $Q_n(z) = 0$.

The numerical experiments show that if $Q_n(z) = 0$ for some n, then for arbitrary $m \neq n$ we have $Q_m(z) \neq 0$, therefore the defect numbers may only be equal zero or one.

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Initiating a new trend in complex equations studying solutions in a given domain: problems, approaches, results

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Abstract. There is a huge number of investigations in complex differential equations when the solutions are meromorphic in the complex plane. The main attention was paid to the value distribution type phenomena of the solutions, particularly to the zeros (more generally to the a-points) of these solutions.

Meantime there are very few studies of meromorphic solutions in a given domain, particularly zeros of similar solutions weren't touched at all.

We initiate studies of meromorphic solutions in a given complex domain: we pose some new problems and give some approaches for their solutions.

Clearly for similar studies we should have some tolls that are valid for large classes of functions in a domain. As some tools we make use results related to three comparatively recent topics, Gamma-lines, proximity property and universal version of value distribution theory; all they are valid for any meromorphic function in a given domain.

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Критерии почти нильпотентности для групп гомеоморфизмов прямой и окружности. Структурные теоремы

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Аннотация. Для класса конечно порожденных групп гомеоморфизмов прямой и окружности установлен критерий почти нильпотентности. В иных терминах, для класса конечно порожденных групп диффеоморфизмов прямой и окружности с взаимно трансверсальными элементами также устанавливаются критерии почти нильпотентности. Более того, для таких групп получены структурные теоремы. При доказательстве структурных теорем ключевыми являются факт наличия, либо отсутствия инвариантной меры, ранее полученный критерий существования инвариантной меры, а также его переформулировки в терминах различных характеристик группы (топологических, алгебраических, комбинаторных). Обсуждается вопрос о типичности как ряда свойств отмеченных характеристик, так и факта существования инвариантной меры для групп диффеоморфизмов прямой и окружности.

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Бегущие волны и функционально дифференциальные уравнения точечного типа. Что общего?

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Аннотация. Одной из центральных задач при изучении функционально-дифференциальных уравнений точечного типа является задача Коши

$$\dot{x}(t) = g(t, x(q_1^*(t)), \ldots, x(q_s^*(t))), \quad t \in \mathbb{R} \tag{1}$$
$$x(\bar{t}) = \bar{x}, \quad \bar{t} \in \mathbb{R}, \quad \bar{x} \in \mathbb{R}^n, \tag{2}$$

где $g : \mathbb{R} \times \mathbb{R}^{ns} \longrightarrow \mathbb{R}^n$ – отображение класса $C^{(0)}$; $q_j^*(.)$, $j = 1, \ldots, s$ – диффеоморфизмы прямой, сохраняющие ориентацию, со свойством

$$h_j < +\infty, \quad h_j = \sup_{t \in \mathbb{R}} |t - q^*(t)|, \quad j = 1, 2, \ldots, s.$$

В исследовании задачи Коши (1)-(2) весьма важна роль группы $Q^* = < q_1^*, \ldots, q_s^* >$. Рассмотрим полное прямое произведение

$$K_Q^n = \prod_{q \in Q} R_q, \quad R_q = \mathbb{R}^n, \quad q \in Q$$

со стандартной тихоновской топологией и элементами $\varkappa = \{x_q\}_{q \in Q}$ в виде бесконечных последовательностей. Для каждого $\bar{q} \in Q$ в пространстве бесконечных последовательностей K_Q^n определен сдвиг $T_{\bar{q}}$ (линейное невырожденное отображение) по правилу

$$T_{\bar{q}} : K_Q^n \to K_Q^n, \quad \bar{q} \in Q, \quad T_{\bar{q}}\{x_q\}_{q \in Q} = \{x_{q\bar{q}}\}_{q \in Q},$$

а также отображение

$$G : \mathbb{R} \times K_Q^n \to K_Q^n,$$
$$\big(G(t, \varkappa)\big)_q = g_q(t, \varkappa) = \dot{q}(t) g\big(q(t), x_{q_1^* q}, \ldots, x_{q_s^* q}\big), \quad q \in Q.$$

Определим бесконечномерное дифференциальное уравнение

$$\dot{\varkappa}(t) = G(t, \varkappa), \quad t \in \mathbb{R}, \quad \varkappa \in K_Q^n, \tag{3}$$
$$\varkappa(\bar{q}(t)) = T_{\bar{q}} \varkappa(t), \quad \bar{q} \in Q \tag{4}$$

с нелокальными ограничениями (4). Решения такой системы называются решениями типа бегущей волны.

Теорема 1. *Каждому решению* $x(t), t \in \mathbb{R}$ *исходного функционально-дифференциального уравнения* (1) *соответствует решение* $\varkappa(t) = \{x_q(t)\}_{q \in Q}, t \in \mathbb{R}$ *системы* (3)-(4) *(решение типа бегущей волны) и наоборот. Такие решения связаны соотношениями* $x_q(t) = x(q(t)), q \in Q$. ■

Будут обсуждаться ряд более общих конструкций, связанных с решениями типа бегущей волны.

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On Noethericity of differential operators in anisotropic spaces

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This paper is devoted to research on Noethericity of differential operators, acting in anisotropic Sobolev spaces in $\mathbb{R}^n$. It is continuation of the authors' works [1, 2, 3]. In this work we study a priori estimates for differential operators in anisotropic spaces, solvability conditions of corresponding equations, necessary and sufficient conditions for Noethericity of the special classes of semielliptical operators. Noethericity of semielliptical operators is also studied in [4, 5].
Consider differential form

$$P\left(x,\mathbb{D}\right)=\sum_{(\alpha:\nu)\leq s}a_{\alpha}(x)D^{\alpha}, \tag{1}$$

where $n,s\in\mathbb{N},\alpha\in\mathbb{Z}_{+}^{n},\nu\in\mathbb{N}^{n}$ ($\mathbb{Z}_{+}^{n}$ – set of n-dimensional multi-indices, $\mathbb{N}^{n}$ – set of multi-indices with natural components), $(\alpha:\nu)=\frac{\alpha_1}{\nu_1}+...+\frac{\alpha_n}{\nu_n}$, $D^{\alpha}=D_1^{\alpha_1}...D_n^{\alpha_n}$, $D_k=i^{-1}\frac{\partial}{\partial x_k}, x=(x_1,...,x_n)\in\mathbb{R}^n, a_{\alpha}(x)$ are sufficiently smooth functions.
For $k\in\mathbb{Z}_{+},\nu\in\mathbb{N}^{n}$ denote by $H^{k,\nu}(\mathbb{R}^n)$ the space of measurable functions $\{u\}$ equipped with the norm

$$\|u\|_{k,\nu}=\sum_{(\alpha:\nu)\leq k}\|D^{\alpha}u\|_{L_2(\mathbb{R}^n)}<\infty. \tag{2}$$

Let $q(x)$ be positive function such that $\frac{1}{q(x)}\rightrightarrows 0$ when $|x|\to\infty$. Denote by $H_q^{k,\nu}(\mathbb{R}^n)$ the space of measurable functions $\{u\}$ equipped with the norm

$$\|u\|_{k,\nu,q}=\sum_{(\alpha:\nu)\leq k}\left\|D^{\alpha}u\cdot q^{(k-(\alpha:\nu))}\right\|_{L_2(\mathbb{R}^n)}<\infty. \tag{3}$$

Denote by $(P;H^{k,\nu})$ and $(P;H_q^{k,\nu})$ operators defined by differential form $P(x,\mathbb{D})$ (see (1)) acting, correspondingly, from $H^{k+s,\nu}(\mathbb{R}^n)$ to $H^{k,\nu}(\mathbb{R}^n)$ and from $H_q^{k+s,\nu}(\mathbb{R}^n)$ to $H_q^{k,\nu}(\mathbb{R}^n)$.
With the certain rate at infinity of the coefficients of the differential form $P(x,\mathbb{D})$ criteria of Noethericity is obtained for operators $(P;H^{k,\nu})$

$(P; H_q^{k,\nu})$. The special a priori estimates are obtained in weighted anisotropic spaces and necessary conditions are established for them. Using these results index equality to zero is proved for the special class of semielliptical operators.

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Sufficient conditions for a balanced bipartite digraph to be even pancyclic

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We consider directed graphs (digraphs) without loops and multiple arcs. Theminology and notation not described below follows [2]. Let D be a digraph of order n. Bondy suggested (see [6] by Chvátal) the following metaconjecture:

Metaconjecture (Bondy). *Almost any non-trivial condition of a undirected graph (digraph) which implies that the graph (digraph) is Hamiltonian also implies that the undirected graph (digraph) is pancyclic. (There may be a "simple" family of exceptional graphs (digraphs)).*

There are various sufficient conditions for a digraph (undirected graph) to be Hamiltonian are also sufficient for the digraph (undirected graph) to be pancyclic unless some extremal cases which are characterized (see, e.g., [1, 2, 4, 6, 8, 9]). In [3], it was proved the following theorem.

Theorem 1 (Bang-Jensen, Gutin, Li [3]). *Let D be a strongly connected digraph of order $n \geq 2$. Suppose that $min\{d(x), d(y)\} \geq n-1$ and $d(x)+d(y) \geq 2n-1$ for any pair of non-adjacent vertices x, y with a common in-neighbour. Then D is Hamiltonian.*

An analogous of Theorem 1 for bipartite digraphs was given by Wang [10], and recently strengthened by the author [7].

Theorem 2 (Wang [10]). *Let D be a strongly connected balanced bipartite digraph of order $2a$, where $a \geq 1$. Suppose that, for every pair of vertices $\{x, y\}$ with a common out-neighbour, either $d(x) \geq 2a-1$ and $d(y) \geq a+1$ or $d(y) \geq 2a-1$ and $d(x) \geq a+1$. Then D is Hamiltonian.*

Definition 1. *Let $D(8)$ be the bipartite digraph with partite sets $X = \{x_0, x_1, x_2, x_3\}$ and $Y = \{y_0, y_1, y_2, y_3\}$, and $A(D(8))$ contains exactly the arcs y_0x_1, y_1x_0, x_2y_3, x_3y_2 and all the arcs of the following 2-cycles: $x_i \leftrightarrow y_i$, $i \in [0, 3]$, $y_0 \leftrightarrow x_2$, $y_0 \leftrightarrow x_3$, $y_1 \leftrightarrow x_2$ and $y_1 \leftrightarrow x_3$.*

Theorem 3 (Darbinyan [7]). *Let D be a strongly connected balanced bipartite digraph of order $2a$, where $a \geq 4$. If D $max\{d(x), d(y)\} \geq 2a-1$ for every pair of vertices $\{x, y\}$ with a common out-neighbour, then D is Hamiltonian unless D is isomorphic to the digraph $D(8)$.*

Motivated by the Bondy metaconjecture, we set the following:
Problem. *Characterize those digraphs which satisfy the conditions of Theorem 3 (or 2) but are not even pancyclic.*

In this note we prove the following theorems.

Theorem 4. *Let D be a strongly connected balanced bipartite digraph of order $2a \geq 8$. If D contains a cycle of length $2a-2$ and $max\{d(x), d(y)\} \geq 2a-2$ for every pair of vertices $\{x, y\}$ with a common out-neigbour, then D contains cycles of every length $2k$, $1 \leq k \leq a-1$.*

Theorem 5. *Let D be a strongly connected balanced bipartite digraph of order $2a \geq 8$. If D is not a directed cycle and $max\{d(x), d(y)\} \geq 2a-1$ for every pair of vertices $\{x, y\}$ with a common out-neighbour, then the following holds: (i) D contains a cycle of length $2a-2$; (ii) D contains cycles of all even lengths less than equal to $2a$ or D is isomorphic to the digraph $D(8)$.*

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A discrete analog of Ramanujan's method

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Solution of equation is one of the oldest and principal problems of mathematics. Bisection, chords, secant and Newton-Raphson methods are classical tools. Less known is Ramanujan method of solution of a certain class of equations

$$f(z) = 0 \tag{1}$$

with analytic functions satisfying the condition $f(0) \neq 0$ and having only one root z_0 having the least modulus. In this case the function

$$g(z) = \frac{1}{f(z)}$$

will be analytic in a neighborhood of 0 and let

$$g(z) = \sum_{n=0}^{\infty} c_n z^n$$

be its Mac-Laurin series expansion.

According to ([1], p. 42) "Ramanujan's discourse is characteristically brief; he ... claims, with no hypotheses, that $\frac{c_{n-1}}{c_n}$ approaches a root of (1)".

Ramanujan method is generalized in [2].

Let f be analytic in some domain $\mathcal{D}$ function, having a finite number of (not obligatory simple) zeros in each bounded subset of $\mathcal{D}$. Let $z \in \mathcal{D}, f(z) \neq 0$. Denote

$$P_0(z) = 1/f(z) \text{ and } P_n(z) = \left(d^n \frac{1}{f}\right)(z), n \in \mathbb{N}, \tag{2}$$

where d is the differentiation operator.

Theorem. Let P_0 be meromorphic in $\mathcal{D}, z \in \mathcal{D}, f(z) \neq 0$. The formula

$$a = z + \lim_{n\to\infty} n \frac{P_{n-1}(z)}{P_n(z)} \tag{3}$$

defines the nearest to z zero of f. If there are many such zeros, then a is the zero of the highest order. The limit does not exist if there are many concurrent zeros of the highest order in the same distance from z.

Formula (3) is hard to implement, as it requires successive calculation of derivatives. Below we propose more realistic method of solution. Let $\{z_0, z_1, \ldots, z_p\}$ be a set of complex numbers and $G = \{g(z_0), g(z_1), \ldots, g(z_p)\}$. Divided differences are defined inductively

$$g[z_0] = g(z_0), \; g[z_0, z_1, \cdots, z_k] = \frac{g[z_1, \cdots, z_k] - g[z_0, \cdots, z_{k-1}]}{z_k - z_0}. \quad (4)$$

Denote by D_k the $k-$th divided difference (4) for the function $g = 1/f$. Define a sequence $\{z_n\}_0^\infty$ by the following formula

$$z_{n+1} = z_n + \frac{D_{n-1}}{D_n}, n \in \mathbb{N}. \quad (5)$$

The initial values z_0 and z_1 are chosen as close as possible to the root to be found. Note that for $n = 1$ this formula coincides with the secant method.

Table 1: Approximate roots of equation $f(x) = 0$

Iteration	$J_0(x)$	$\tan x - x$	$\sin x - x + 0.5$
0	2	4	1
1	3	5	2
2	2.462638992987265	3.486817237244572	1.366316960917713
3	2.402495017462122	4.679485154090827	1.506993668228667
4	2.404824149524840	4.495488676902464	1.497332091360431
5	2.404825557694929	4.493406165121571	1.497300389340039
6	2.404825557695773	4.493409457914472	1.497300389095892
7	2.404825557695773	4.493409457909064	1.497300389095893

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Geodesic flows on the hypersurface of the energy of a three-body system

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Introduction. In the recent work [1] it is proved that the three-particle classical problem in the most general case is equivalent to the problem of geodesic flows on a Riemannian manifold. As shown, the formulation of the problem on the curved space allows to reveal the hidden internal symmetries of the dynamic system, which helps to achieve a more complete reduction of the problem. Namely, within the framework of the new representation, the three-body problem instead of the system of the 8th order is described by the system of the 6th order.

In this article, we consider the problem of three bodies under the influence of the environment, taking into account that the influence of the environment on the body system has both regular and stochastic impacts. A new type of second-order partial differential equations describing geodesic flows on a Riemannian manifold is derived. It is proved that the timing parameter in this equation branches during the evolution that making the equation irreversible relative to it.

Section 1. The classical three-body problem in general case reduces to the system of the $6th$ order [1]:

$$\begin{aligned}
&\dot{\xi}^1 = A^1(\{\bar{x}\},\{\bar{\xi}\}), \qquad \xi^1 = \dot{x}^1,\\
&\dot{\xi}^2 = A^2(\{\bar{x}\},\{\bar{\xi}\}), \qquad \xi^2 = \dot{x}^2,\\
&\dot{\xi}^3 = A^3(\{\bar{x}\},\{\bar{\xi}\}), \qquad \xi^3 = \dot{x}^3, \qquad \dot{\xi}^i = d\xi^i/ds,
\end{aligned} \tag{1}$$

where $\{\bar{\xi}\} = (\xi^1,\xi^2,\xi^3)$ and $\{\bar{x}\} = (x^1,x^2,x^3)$, in addition, the following designations are made:

$$A^1(\{\bar{x}\},\{\bar{\xi}\}) = a_1\big\{(\xi^1)^2 - (\xi^2)^2 - (\xi^3)^2 - \Lambda^2\big\} + 2\xi^1\big\{a_2\xi^2 + a_3\xi^3\big\},$$

$$A^2(\{\bar{x}\},\{\bar{\xi}\}) = a_2\big\{(\xi^2)^2 - (\xi^3)^2 - (\xi^1)^2 - \Lambda^2\big\} + 2\xi^2\big\{a_3\xi^3 + a_1\xi^1\big\},$$

$$A^3(\{\bar{x}\},\{\bar{\xi}\}) = a_3\big\{(\xi^3)^2 - (\xi^1)^2 - (\xi^2)^2 - \Lambda^2\big\} + 2\xi^3\big\{a_1\xi^1 + a_2\xi^2\big\}.$$

In (1) $s = s(\{\bar{x}\})$ is the *timing parameter*, $\{\bar{x}\} = (x^1,x^2,x^3) \in \mathcal{M}_t$ and $\mathcal{M}_t$ is the tangent bundle of the 3-dimensional Riemannian manifold

$\mathcal{M}^{(3)}$, which has a conformally Euclidean metric; $g_{ij}(\{\bar{x}\}) = g(\{\bar{x}\})\delta_{ij}$, where $g(\{\bar{x}\}) = [E - U(\{\bar{x}\})] > 0$, E and $U(\{\bar{x}\})$ are the total energy and total interaction potential of bodies system, respectively, δ_{ij} *is the Kronecker delta function.* In equations system (1), $a_i(\{\bar{x}\}) = -(1/2)\partial_{x^i} \ln g(\{\bar{x}\})$ and $\Lambda(\{\bar{x}\}) = Jg^{-1}(\{\bar{x}\})$, where $J = const$ is the total angular momentum.

Let external random forces influence the three-body system. Then the dynamical system, in particular, can be described by stochastic equations of the Langevin type:

$$\dot{\chi}^\mu = A^\mu(\{\chi\}) + \eta^\mu(s), \qquad \{\chi\} = (\{\bar{\xi}\}, \{\bar{x}\}), \quad \mu = \overline{1,6}, \tag{2}$$

where $A^\mu(\{\chi\})$ are regular functions (1), while $\eta^\mu(s)$ - random functions.

Definition 1. *The joint probability density for the independent variables can be represented in the form:*

$$P(\{\chi\}, s) = \prod_{\mu=1}^{6} \big\langle \delta\big[\chi^\mu(s) - \chi^\mu\big]\big\rangle. \tag{3}$$

Theorem 1. *If the random functions $\eta^\mu(s)$ satisfy the following correlation relations:*

$$\langle \eta^\mu(s) \rangle = 0, \qquad \langle \eta^\mu(s)\eta^\mu(s') \rangle = 2\epsilon\delta(s - s'), \qquad \epsilon = const, \tag{4}$$

then, using the equations (2), it is possible to obtain the equation of joint probability distribution for geodesic trajectories in the phase space:

$$\frac{\partial P}{\partial s} = \sum_{\mu=1}^{6} \frac{\partial}{\partial \chi^\mu}\Big[A^\mu(\{\chi\}) + \epsilon\frac{\partial}{\partial \chi^\mu}\Big]P. \tag{5}$$

Note that this equation is not an ordinary equation, because the *timing parameter* $s \in \mathcal{M}^{(3)}$ branches during the evolution of dynamical system, making the equation (5) an irreversible with respect to "s". Moreover, this equation is not a Cauchy problem because of the branching of the timing parameter. The latter generates new geodesic flows with different topological features, which can further evaluates to various asymptotic subspaces of the 6-dimensional phase space. An expression is also obtained for the probabilities of transitions between different asymptotic scattering channels. More details about the work can be found in [1].

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The classical three-body problem and the Poincaré conjecture

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Introduction. As shown [1], the classical three-body problem in the most general case can reduced to the system of $6th$ order, if it to formulate as a problem of geodesic flow on the Riemannian manifold $\mathcal{M} \cong \mathcal{M}^{(3)} \times \mathcal{S}_t^3$, where $\mathcal{M}^{(3)}$ is the conformally Euclidean space defined by the metric $g_{ij} = g(\{\bar{x}\})\delta_{ij}$, while δ_{ij} denotes the Kronecker symbol, $\{\bar{x}\} \equiv (x^1, x^2, x^3) \in \mathcal{M}_t^{(3)}$ is the set of coordinates on the tangential bundle, $\mathcal{S}_t^3 \ni (x^4, x^5, x^6) \equiv \{\underline{x}\}$ is the local rotation group $SO(3)$. In addition, $g(\{\bar{x}\}) = [E - U(\{\bar{x}\})] > 0$, where E and $U(\{\bar{x}\})$ the total energy and the full interaction potential between bodies, respectively. In proving the equivalence of the problem of geodesic flow on the Riemannian manifold and the original Newtonian three-body problem a key role is played the homomorphism theorem between of the Euclidean subspace $\mathbb{E}^6 \subset \mathbb{R}^6$ and the six-dimensional manifold $\mathcal{M}$, which is an essential generalization of the Poincaré conjecture.

Theorem. *The Euclidean subspace $\mathbb{E}^3 \subset \mathbb{R}^3$, in which relative movements of classical bodies occur, is homeomorphic to the three-dimensional manifold $\mathfrak{S}^{(3)}$, generated by the underdetermined system of algebraic equations:*

$$
\begin{aligned}
\alpha_1^2 + \beta_1^2 + \gamma_1^2 &= g(\{\bar{x}\}), \qquad \alpha_1\alpha_2 + \beta_1\beta_2 + \gamma_1\gamma_2 = 0,\\
\alpha_2^2 + \beta_2^2 + \gamma_2^2 &= g(\{\bar{x}\}), \qquad \alpha_1\alpha_3 + \beta_1\beta_3 + \gamma_1\gamma_3 = 0,\\
\alpha_3^2 + \beta_3^2 + \gamma_3^2 &= g(\{\bar{x}\}), \qquad \alpha_2\alpha_3 + \beta_2\beta_3 + \gamma_2\gamma_3 = 0,
\end{aligned}
\tag{1}
$$

that in its turn is homeomorphic to the three-dimensional manifold $\mathcal{M}^{(3)}$.

Proof. Let us consider a linear infinitesimal element (ds) in both coordinate systems $\{\rho\} = (\rho_1, ..., \rho_6) \in \mathbb{E}^6$ and $\{x\} = (\{\bar{x}\}, \{\underline{x}\}) \in \mathcal{M}$. Equating them, we can write:

$$
(ds)^2 = \gamma^{\alpha\beta}(\{\rho\})d\rho_\alpha d\rho_\beta = g_{\mu\nu}(\{\bar{x}\})dx^\mu dx^\nu, \quad \alpha, \beta, \mu, \nu = \overline{1,6}, \tag{2}
$$

from which one can find the following system of algebraic equations:

$$\gamma^{\alpha\beta}(\{\rho\})\rho_{\alpha;\mu}\rho_{\beta;\nu} = g_{\mu\nu}(\{\bar{x}\}) = g(\{\bar{x}\})\delta_{\mu\nu}, \quad \rho_{\alpha;\mu} = \partial\rho_\alpha/\partial x^\mu. \tag{3}$$

Recall that the derivatives $\rho_{\alpha;\mu}$ define coordinate transformations from $\{\rho\}$ to $\{x\}$ (conditionally we will call direct transformations).

From (2), in a similar way, we obtain the system of algebraic equations for the derivatives that determine inverse transformations:

$$\gamma^{\alpha\beta}(\{\rho\})\delta_{\alpha\bar{\alpha}}\delta_{\beta\bar{\beta}}g^{-1}(\{\bar{x}\}) = x^{\mu}_{;\,\bar{\alpha}}x^{\nu}_{;\,\bar{\beta}}\,\delta_{\mu\nu}, \qquad \bar{\alpha},\bar{\beta} = \overline{1,6}, \tag{4}$$

where $x^{\mu}_{;\,\alpha} = \partial x^\mu/\partial\rho^\alpha$.

At first we consider the system of equations (3), which is related to direct coordinate transformations. It is not difficult to see that the system of algebraic equations (3) is underdefined with respect to the variables $\rho_{\alpha;\mu}$, since it consists of 21 equations, while the number of unknown variables is 36. Obviously, when these equations are compatible, then the system (3) has an infinite number of real and complex solutions. Note that for the classical three-body problem, the real solutions of the system (3) are important, which at the general examination form a 15-dimensional manifold. Since the tensor $g_{\mu\nu}(\{\bar{x}\})$ is still defined in a rather arbitrary way we can impose additional conditions on it in order to determine the minimal dimension of the manifold allowing a separation of the base $\mathcal{M}^{(3)}$ from the layer S^3_t.

Let us make a new designations:

$$\alpha_\mu = \rho_{1;\mu},\ \beta_\mu = \rho_{2;\mu},\ \gamma_\mu = \rho_{3;\mu},\ u_\mu = \rho_{4;\mu},\ v_\mu = \rho_{5;\mu},\ w_\mu = \rho_{6;\mu}. \tag{5}$$

Taking into account the fact that the tensor $g_{\mu\nu}(\{\bar{x}\})$ still fairly general one, we can require fulfillment for its elements the following conditions:

$$\begin{aligned} &\alpha_4 = \alpha_5 = \alpha_6 = 0, \quad \beta_4 = \beta_5 = \beta_6 = 0, \quad \gamma_4 = \gamma_5 = \gamma_6 = 0, \\ &u_1 = u_2 = u_3 = 0, \quad v_1 = v_2 = v_3 = 0, \quad w_1 = w_2 = w_3 = 0. \end{aligned} \tag{6}$$

Using (5) and conditions (6) from the equation (3) we can obtain two independent underdetermined systems of algebraic equations (six equations nine unknowns), the first of which is the system (1), while the second is defined in each point of $\{\bar{x}\}_i \in \mathcal{M}^{(3)}$ and is related to the set of derivatives of *external coordinates* (see (5)). The manifold $\mathfrak{S}^{(3)}$ is in a one-to-one mapping on the one hand with the subspace $\mathbb{E}^3 \in \{\bar{\rho}\} = (\rho_1, \rho_2, \rho_3)$ and on the other hand with the manifold $\mathcal{M}^{(3)}$. This assertion follows from the fact that all points of the manifold $\mathcal{M}^{(3)}$ and the *internal space* $\mathbb{E}^3$ are pairwise connected through the corresponding derivatives, which, as unknown variables, enter the algebraic equations system (1). Lastly, given the fact, that there are also inverse coordinate transformations, it can be argued that $\mathfrak{S}^{(3)}$ is homeomorphic as to $\mathbb{E}^3$ as well as $\mathcal{M}^{(3)}$. From this follows, that $\mathbb{E}^3$ and $\mathcal{M}^{(3)}$ *are homeomorphic* too.

Theorem is proved.

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Comparison of power of generalized-homogeneous polynomials

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Definition 1 (see [3] or [4]). *We will say that a polynomial P is more powerful than a polynomial Q and write as $P > Q$ (or $Q < P$), if there exists a constant $c > 0$ such that*

$$|Q(\xi)| \leq c[|P(\xi)| + 1] \qquad \forall \xi \in \mathbb{R}^n.$$

Definition 2. *Let $\lambda \in \mathbb{R}^n$. A polynomial $R(\xi)$ is called λ–homogeneous (generalized homogeneous) of λ – order $d_R = d_R(\lambda)$ if $P(t^\lambda \xi) :=$ $P(t^{\lambda_1} \xi_1, \dots, t^{\lambda_n} \xi_n) = t^d \, P(\xi)$ for all $\xi \in \mathbb{R}^n$ and for any $t > 0$.*

For a λ–homogeneous polynomial $R(\xi)$ we denote:
$\Sigma(R) := \{\eta \in \mathbb{R}^n, \ |\eta| = 1, \ R(\eta) = 0\}$, $\mathfrak{A}(R) := \{\alpha \in \mathbb{N}_0^n : D^\alpha R(\eta) \neq 0\}$ and $\Delta(R) := \min\limits_{\alpha \in \mathfrak{A}(R)} |\alpha|$.

An interesting application of the operator comparison is the question of adding lowest terms to a given (elliptic, hypoelliptic, hyperbolic etc. (see, for example, [1]-[5])) operator that does not violate its character, i.e. preserves its ellipticity, hypoellipticity etc. For example, in [4] the following results are proved:

1) Let P be a hypoelliptic polynomial of order d_0, R be a λ–homogeneous polynomial of λ–order $d(Q) < d_0$ and $R < P_0$, then $P + R$ is also hypoelliptic;

2) If a polynomial P, with in general complex coefficients, is hypoelliptic and $Q < P$ then there exists a number $\varepsilon > 0$ such that for any complex number $a : |a| < \varepsilon$ the polynomial $P + a\,Q$ is also hypoelliptic.

This and other examples, the number of which can be multiplied, show the relevance of the problem of finding polynomials Q having a lower power then a given (in particular, a generalized–homogeneous) polynomial P.

The purpose of this report is the following.

Theorem. *Let P and Q be λ–homogeneous polynomials λ–orders d_P and d_Q respectively, with $P > Q$. Then*
1) $d_P \geq d_Q$,
2) $\Sigma(Q) \supset \Sigma(P)$,

3) $\Re(Q) \subset \Re(P)$,
4) *for each point* $\eta \in \Sigma(P)$ *there exists a neighborhood* $U(\eta)$ *and a constant* $c > 0$ *such that*

$$|Q(\xi)| \leq c\,|P(\xi)|^{\frac{d_Q}{d_P}} \quad \forall \xi \in U(\eta) \tag{1}$$

5)

$$\frac{d_Q}{d_P} \leq \frac{\Delta(\eta, Q)}{\Delta(\eta, P)} \quad \forall \eta \in \Sigma(P) \tag{2}$$

6) $Q < P^{\frac{d_Q}{d_P}}$, *i.e there is a number* $c > 0$ *such that*

$$|Q(\xi)| \leq c\,[1 + |P(\xi)|^{\frac{d_Q}{d_P}}] \qquad \forall \xi \in \mathbb{R}^n. \tag{3}$$

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The description of semigroups, which polynomially satisfy weakly associative $\{3\}$-hyperidentities

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The present talk is devoted to the necessary and sufficient conditions of semigroups, which polynomially satisfy the following hyperidentities (1)-(3).

Theorem 1. *The semigroup $Q(\cdot)$ polynomially satisfies the following hyperidentity*

$$F\left(F\left(x,x,x\right),x,x\right)=F\left(x,x,F\left(x,x,x\right)\right) \tag{1}$$

iff $Q(\cdot)$ is a semigroup with the identity:

$$x^3=x^2.$$

Theorem 2. *The semigroup $Q(\cdot)$ polynomially satisfies the following hyperidentity*

$$F\left(F\left(x,x,x\right),x,x\right)=F\left(x,x,F\left(x,y,x\right)\right) \tag{2}$$

iff $Q(\cdot)$ is a semigroup with the identities:

$$\begin{cases} x^2y=xy, \\ xy^2=xy \end{cases}$$

Theorem 3. *The semigroup $Q(\cdot)$ polynomially satisfies the following hyperidentity*

$$F\left(F\left(x,x,x\right),x,y\right)=F\left(x,x,F\left(x,x,y\right)\right) \tag{3}$$

iff $Q(\cdot)$ is a semigroup with the identities:

$$\begin{cases} x^2y=xy, \\ xy^2=xy \end{cases}$$

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Monotone maps on matrices are invertible

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This talk is based on series of joint works with G. Dolinar, M. Efimov and J. Marovt.

The first result on transformations preserving matrix invariants is due to Frobenius. This result describes the structure of linear maps T preserving the determinant function, i.e., $\det X = \det T(X)$ for all X. Later there were several extension of this result which are due to Dieudonné, Schur, Dynkin and others. Different methods and techniques are used to obtain these results. Along the same lines, there was an intensive investigation of maps preserving order relations on operator and matrix algebras during the past decades.

There are many order relations on matrices. Some of them are originated from semigroup theory, for example, the orders by Hartwig, Nambooripad and Drazin. Order relations on matrices are important for theoretical studies and applications.

Monotone transformation with respect to a particular order relation is a map which preserves this order. We show that surjective monotone additive transformations on matrices with respect to several orders are automatically bijective, investigate their properties and provide a complete characterization of such transformations.

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On Geometry of Submanifolds in Pseudoeuclidean Space E_n^{2n}

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The present work is devoted to the study of one class of submanifolds in pseudoeuclidean space E_n^{2n}. This class of submanifolds $M \subset E_n^{2n}$ of dimension $2m$ satisfying condition $2m > n$ is determined by relations

$$\omega^{m+i} = \omega_i, \quad \omega_{m+i} = 0, \qquad i = 1, 2, \ldots, n-m. \tag{1}$$

The structure equations of the total space E_n^{2n} can be reduced to the form

$$\begin{array}{ll} d\omega^I = \omega_K^I \wedge \omega^K, & d\omega_I = -\omega_I^K \wedge \omega_K, \\ d\omega_K^I = \omega_P^I \wedge \omega_K^P, & I, K, P = 1, 2, \ldots, n. \end{array} \tag{2}$$

We'll use the following indexation here: $i, k = 1, \ldots, n-m$; $a, b = n-m+1, \ldots, m$. The metric form of the total pseudoeuclidean Rashevsky space E_n^{2n} defined by the invariant bilinear close nondegenerate form $d\varphi = \omega^I \wedge \omega_I$ induces the bilinear form

$$d\varphi^* = \omega^i \wedge \omega_i + \omega^a \wedge \omega_a. \tag{3}$$

Relations (1) are identities on M. Exterior differentiation of these identities and applications of general structure equations come to the following two identities:

$$\begin{array}{c} \left(\omega_{m+k}^{m+i} + \omega_i^k\right) \wedge \omega_k + \omega_i^a \wedge \omega_a + \omega_k^{m+i} \wedge \omega^k + \omega_a^{m+i} \wedge \omega^a = 0, \\ \omega_{m+i}^k \wedge \omega_k + \omega_{m+i}^a \wedge \omega_a = 0. \end{array} \tag{4}$$

If the submanifold M has the structure of double fiber bundle then $\omega_{m+k}^i \wedge \omega_k = 0$ and $\omega_{m+k}^a \wedge \omega_k = 0$. The second identity comes to the form $\omega_{m+i}^a \wedge \omega_a = 0$ and we can see that the secondary forms ω_{m+i}^a are linear combinations of basic principal forms $\omega_{n-m+1}, \ldots, \omega_m$ only. Other side the conditions $\omega_{m+k}^a \wedge \omega_k = 0$ show that these forms have nontrivial expansions by forms $\omega_1, \ldots, \omega_{n-m}$ only. It's possible if and only if $\omega_{m+i}^a = 0$. So we obtain the expansion $\omega_{m+i}^k = C_{m+i}^{kp} \omega_p$, $C_{m+i}^{kp} = C_{m+i}^{pk}$. Application of Cartan's lemma to the first identity from (4) gives the expansions for secondary forms $\omega^{m+i}_{m+k}+\omega^k_i$, ω^a_i, ω^{m+i}_k, ω^{m+i}_a. Substituting these expansions back in the first identity from (4) we'll obtain some relations for coefficients and using them we'll come to the following equalities:

$$\begin{aligned}
&\omega^{m+i}_{m+k}+\omega^k_i = C^k_{ip}\omega^p + C^k_{ia}\omega^a + C^{kp}_i\omega_p + C^{ka}_i\omega_a,\\
&\qquad\qquad\qquad\qquad\qquad\qquad C^{kp}_i = C^{pk}_i,\ C^k_{ip}=C^k_{pi};\\
&\omega^a_i = C^a_{ik}\omega^k + C^a_{ib}\omega^b + C^{ak}_i\omega_k + C^{ab}_i\omega_b,\ C^a_{ik}=C^a_{ki},\ C^{ab}_i = C^{ba}_i;\\
&\omega^{m+i}_k = C^{m+i}_{kp}\omega^k + C^{m+i}_{ka}\omega^a + C^p_{ik}\omega_p + C^a_{ik}\omega_a,\ C^{m+i}_{kp}=C^{m+i}_{pk};\\
&\omega^{m+i}_a = C^{m+i}_{ka}\omega^k + C^{m+i}_{ab}\omega^b + C^k_{ia}\omega_k + C^b_{ia}\omega_b,\ C^{m+i}_{ab}=C^{m+i}_{ba}.
\end{aligned} \tag{5}$$

Substitution of relations $\omega^a_{m+i}=0$ in the general structure equations comes to the conditions

$$\omega^a_p\wedge\omega^p_{m+k}=0 \quad \text{or} \quad C^{pt}_{m+k}\omega^a_p\wedge\omega_t=0. \tag{6}$$

We'll study the case when for at least one value of the index i $(=1,\dots,n-m)$ the matrix (C^{pt}_{m+k}) is nondegenerate: $\det(C^{pt}_{m+k})\neq 0$. Using the Cartan's lemma we see, that the form $C^{pt}_{m+k}\omega^a_p$ is the linear combination of basic principal forms $\omega_1,\dots,\omega_{n-m}$ only and then applying the conditions $\det(C^{pt}_{m+k})\neq 0$ we come to the conclusion that secondary forms ω^a_i have nontrivial expansions by basic forms $\omega_1,\dots,\omega_{n-m}$ only: $\omega^a_i = C^{ak}_i\omega_k$. Substitution of this expansion in the identity (6) comes to the algebraic relation

$$C^{ip}_{m+k}C^{at}_i = C^{it}_{m+k}C^{ap}_i.$$

The following statement is true.

Theorem. *The metric connection of* $2n$ *dimensional pseudoeuclidean Rashevsky space* E^{2n}_n *induces the structure of double fiber bundle on* $2m$ *dimensional* $(2m>n)$ *submanifold* M *defined by equations* (1) *with special type affine connection on* M *determined by forms* ω^i, ω^a, ω_i, ω_a, ω^i_k, ω^a_b, ω^i_a *and functions* C^{ak}_i, C^{ip}_{m+k} $(i,k=1,\dots,n-m;\ a,b=n-m+1,\dots,m)$, *satisfying structure equation*

$$\begin{aligned}
&d\omega^i = \omega^i_k\wedge\omega^k + \omega^i_a\wedge\omega^a,\ d\omega^a = \omega^a_b\wedge\omega^b + C^{ak}_i\omega_k\wedge\omega^i,\\
&d\omega_i = -\omega^k_i\wedge\omega_k - C^{ak}_i\omega_k\wedge\omega_a,\ d\omega_a = -\omega^k_a\wedge\omega_k - \omega^b_a\wedge\omega_b,\\
&d\omega^i_k = \omega^i_p\wedge\omega^p_k + C^{ap}_k\omega^i_a\wedge\omega_p,\ d\omega^a_b = \omega^a_c\wedge\omega^c_b + C^{ak}_i\omega^i_b\wedge\omega_k,\\
&d\omega^i_a = \omega^i_p\wedge\omega^p_a + \omega^i_b\wedge\omega^b_a,\\
&dC^{ak}_i = C^{ap}_i\omega^k_p + C^{bk}_i\omega^a_b - C^{ak}_p\omega^p_i - C^{ap}_iC^{bk}_i\omega_b + C^{akp}_i\omega_p
\end{aligned}$$

is induced on submanifold M.

Some problems of spectral theory

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First, we investigate the dependence of spectral data of Sturm-Liouville operator on parameters defining the boundary conditions. With this aim we introduce the concept of "Eigenvalues function of family of Sturm-Liouville operators" (EVF) and investigate its properties.
Secondly we solve the inverse Sturm-Liouville problem by EVF.
We also provide an analogue of uniqueness theorem (in inverse problem) of Marchenko and one generalization of theorem of Ambarzumian.
New uniqueness theorems we also prove in inverse problems for canonical Dirac systems.
We give the description of isospectral Dirac operators.
We have proved, that in common case the analogue of Ambarzumian theorem for Dirac operator is not true, but in the same time, we describe particular cases, when there are analogues of Ambarzumian theorem.
We also give some new results in constructive solution of inverse problem for Dirac system.
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About the solvability of regular hypoelliptic equations in R^n

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In the current work the unique solvability of regular hypoelliptic equations in multianisotropic weighted spaces is proved by means of special integral representation of functions through a regular operator. The existence of the solutions is proved by constructing approximate solutions using multianisotrpic integral operators.The paper presents a generalization of the results obtained by G.V. Demidenko in [1]-[2], where approximate solutions for quasi-elliptic equations are constructed in $\mathbb{R}^n$ by using a special integral representation obtained by S.V. Usepnski in [3]. The difficulty of studying regular hypoelliptic equations lies in the fact that if the parts of elliptic and quasi-elliptic operators with higher order are respectively homogeneous and generalized homogeneous, then the part of the regular operator with the higher order is multi-nonhomogeneous. In order to arrive at the current results, as a matter of fact, we used a special integral representation via multianisotropic kernels and estimates for multi-anisotropic kernels obtained in [4]-[6].

Consider the differential operator

$$P(D) = \sum_{\alpha \in \partial' \mathfrak{N}} a_\alpha D^\alpha \tag{1}$$

with real coefficients a_α. Suppose that the operator $P(D)$ is a regular operator, i.e. there exists a constant number $\chi > 0$, such that for any $\xi \in \mathbb{R}^n$ the following inequality holds:

$$|P(\xi)| = \left| \sum_{\alpha \in \partial' \mathfrak{N}} a_\alpha \xi^\alpha \right| \geq \chi \sum_{\alpha \in \partial' \mathfrak{N}} |\xi^\alpha| \,. \tag{2}$$

For a positive parameter ν and a natural number k denote $G_0(\xi, \nu) = e^{-(\nu P(\xi))^{2k}}$, $G_1(\xi, \nu) = 2k e^{-(\nu P(\xi))^{2k}} (\nu P(\xi))^{2k-1}$ and let $\hat{G}_0(t, \nu)$, $\hat{G}_1(t, \nu)$ be the corresponding Fourier transforms of these functions.

For the function $f \in L_p(\mathbb{R}^n)$ denote (see [4])

$$U_h(x) = \frac{1}{(2\pi)^{\frac{n}{2}}} \int_h^{h^{-1}} d\nu \int_{\mathbb{R}^n} f(t) \int_{\mathbb{R}^n} e^{-i(t-x,\xi)} G_1(\xi, \nu) d\xi dt. \tag{3}$$

Using vertices $\alpha^i : \alpha^i \neq 0$ $(i = 1, \ldots, M)$ of a polyhedron $\mathfrak{N}$ define a multi-anisotropic distance $\rho_{\mathfrak{N}}(x) = \left(\sum_{i=1}^{M} x^{2\alpha^i}\right)^{1/2}$ and weighted spaces $W_{p,\sigma}^{\mathfrak{N}}(\mathbb{R}^n)$, which are the completion of space $C_0^\infty(\mathbb{R}^n)$ by the norm

$$\|U\|_{W_{p,\sigma}^{\mathfrak{N}}(\mathbb{R}^n)} = \sum_{\alpha \in \mathfrak{N}} \left\| (1 + \rho_{\mathfrak{N}}(x))^{-\sigma(1 - \max\limits_i (\mu^i, \alpha))} D_x^\alpha U(x) \right\|_{L_p(\mathbb{R}^n)},$$

where $0 < \sigma < 1$.

Let us study the equation

$$P(D)U = f, \tag{4}$$

where $P(D)$ is the operator (1), which satisfies the regularity condition (2). We prove the following theorem on the existence and uniqueness of a solution of equation (4).

Theorem. Let $|\lambda| > 1$, $\frac{|\lambda|}{p} > \sigma > 1 - |\lambda| + \frac{|\lambda|}{p}$. Then for any function $f \in L_p(\mathbb{R}^n) \bigcap L_{1,-\sigma}(\mathbb{R}^n)$ the equation (4) has a unique solution U from the class $W_{p,\sigma}^{\mathfrak{N}}(\mathbb{R}^n)$, which is the limit in the class $W_{p,\sigma}^{\mathfrak{N}}(\mathbb{R}^n)$ of approximate solutions U_h, defined by (3), as $h \to 0$, and there exists a constant $C > 0$, that for any function $f \in L_p(\mathbb{R}^n) \bigcap L_{1,-\sigma}(\mathbb{R}^n)$ the following estimate holds:

$$\|U\|_{W_{p,\sigma}^{\mathfrak{N}}(\mathbb{R}^n)} \leq C \left(\|f\|_{L_p(\mathbb{R}^n)} + \|f\|_{L_{1,-\sigma}(\mathbb{R}^n)} \right).$$

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О построении неподвижной точки в пространстве l_1 для одной бесконечной системы нелинейных алгебраический уравнений

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Аннотация. Исследуется специальный класс бесконечной системы нелинейных алгебраических уравнений с матрицами Теплица-Ганкеля. Указанный класс уравнений имеет непосредственное применение в теории переноса излучения в спектральных линиях. Доказывается существование покомпонентно положительного решения данной системы в пространстве l_1.

Настоящая работа посвящена изучению вопроса разрешимости в пространстве l_1 следующей бесконечной системы нелинейных алгебраических уравнений:

$$x_n = \sum_{j=0}^{\infty} a_{n-j} h_j \left(x_j\right) + \sum_{j=1}^{\infty} a_{n+j} h_j^* \left(x_j\right), \ n = 0, 1, 2 \dots \tag{1}$$

относительно искомого бесконечного вектора $x = (x_0, x_1, x_2, \dots x_n, \dots)^T$ (T- знак транспонирования).

В системе (1) бесконечные матрицы Теплица и Ганкеля $A \equiv (a_{n-j})_{n,j=0}^{\infty}$ и $B \equiv (a_{n+j})_{n,j=0}^{\infty}$ соответственно удовлетворяют следующим условиям:

$$a_{-j} = a_j\,; \quad \forall\, j \in \mathbb{N} \cup \{0\},\, a_n > 0,\, \forall n \in \mathbb{Z}, \tag{2}$$

$$\sum_{i=-\infty}^{\infty} a_i = 1, \quad \sum_{j=0}^{\infty} j^2 a_j < +\infty, \tag{3}$$

$$a_{n+1} < a_n, \quad \forall n \in \mathbb{N} \cup \{0\}. \tag{4}$$

Система (1) возникает в дискретных задачах теории переноса излучения в спектральных линиях (см.[1-2]). Такие системы встречаются также в кинетической теории газов и в p-адической теории струны (см.[3-5]).

Основным результатом настоящей заметки является следующая:

Теорема. *Пусть существуют числа* $\alpha \in \left[0, \dfrac{1}{2}\right]$ *и* $\eta \in (0,1)$, *такие что*

a) *при всяком фиксированном* $j \in \mathbb{N} \cup \{0\}$ *функции* $h_j(u)$ *и* $h_j^*(u) \uparrow$ *по* u *на отрезке* $[P_j(\eta), 1]$, *где*

$$P_j(\eta) \equiv \eta \sum_{m=j+1}^{\infty} a_m, \quad j \in \mathbb{N} \cup \{0\}, \tag{5}$$

b) $h_j, h_j^* \in C[P_j(\eta), 1], \quad j = 0, 1, 2, \ldots,$

c) *выполняются следующие неравенства*

$$0 \leq h_j(u) \leq 1 - (1-u)^{\alpha},\ u \in [P_j(\eta), 1], \quad j = 0, 1, 2, \ldots, \tag{6}$$

$$h_j^*(P_j(\eta)) \geq \eta,\ h_j^*(1) \leq 1, \quad j = 0, 1, 2, \ldots \tag{7}$$

Тогда, при условиях (2)-(4), *система* (1) *имеет покомпонентно положительное решение в пространстве* l_1, *т.е существует* $x = (x_0, x_1, x_2, \ldots x_n, \ldots)^T$, *координаты которого удовлетворяют системе* (1), *причем* $x_j > 0$, $\forall j \in \mathbb{N} \cup \{0\}$ *и* $\sum\limits_{j=0}^{\infty} x_j < +\infty$.

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The GUE turning point process at turning points of lozenge tilings

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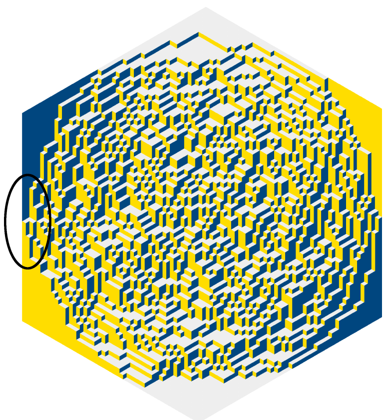
The lozenge tiling model is the model of tilings of planar regions with the three tiles ,  and  called lozenges. There are many different equivalent formulations of the model, including as the dimer model on the hexagonal lattice, as plane partitions, or as Gelfand-Tsetlin patterns.

In the thermodynamic limit of random lozenge tilings the model exhibits the arctic curve phenomenon: there is a certain curve, called the frozen boundary, outside of which the tilings become deterministic (these are the frozen regions), and inside the randomness remains (liquid region). In particular the frozen boundary develops special points where the liquid region meets with two different frozen regions. These are called turning points (see Figure 1).



In a paper titled "The birth of a random matrix" it was conjectured by Okounkov and Reshetikhin [1] that in the scaling limit of the model the local point process near turning points should converge to the Gaussian Unitary Ensemble (GUE) corner process from random matrix theory. We will discuss a joint result with Leonid Petrov establishing the GUE corner process when the underlying measure is the "homogeneous q to the volume" measure. The result can be interpreted as an Interlacing Central Limit Theorem.

We'll also see how the GUE corner process is modified when weights are not homogeneous. The modified process does not correspond to a random matrix model anymore.

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Some domination parameters of graphs and their applications

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Abstract. G is a simple graph with vertex set $V(G)$ and edge set $E(G)$ (briefly V and E). For every vertex $v \in V$, the open neighborhood $N(v)$ is the set $\{u \in V \mid uv \in E\}$ and its *closed neighborhood* is the set $N[v] = N(v) \cup \{v\}$. The *open neighborhood* of a set $S \subseteq V$ is the set $N(S) = \cup_{v \in S} N(v)$, and the *closed neighborhood* of S is the set $N[S] = N(S) \cup S$. $d(x, y)$ denotes the distance between vertices x and y, $\Delta(G)$ is the maximum degree in G.

A set $D \subseteq V$ is a dominating set of G, if for every vertex $v \in V - D$, there exists a vertex $u \in D$ such that v and u are adjacent. The minimum cardinality of a dominating set in G is the domination number denoted $\gamma(G)$. A minimum dominating set of a graph G is called a $\gamma(G)$-set.

A set $D \subseteq V$ is a total dominating set (TDS) of the graph G if each vertex of G has a neighbor in D. Equivalently, a set $D \subseteq V(G)$ is a TDS of a graph G if D is a dominating set of G and $\langle D \rangle$ does not contain an isolate vertex. The cardinality of a minimum TDS in G is the total domination number and is denoted by $\gamma_t(G)$. A minimum TDS of a graph G is called a $\gamma_t(G)$-set.

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A set $S \subseteq V$ is a connected dominating set of G if it is a dominating set and the induced subgraph $\langle S \rangle$ is connected. The minimum cardinality of a connected dominating set in G is the connected domination number denoted $\gamma_c(G)$. A minimum connected dominating set of a graph G is called a $\gamma_c(G)$-set.

A dominating set S is called an independent dominating set of G if S is an independent set. The minimum cardinality among the independent dominating sets of G is the independent domination number, denoted $\gamma_i(G)$. A minimum independent dominating set of a graph G is called a $\gamma_i(G)$-set.

A set S is called a global dominating set of G if S is a dominating set of both G and its complement $\overline{G}$. The global domination number $\gamma_g(G)$ of G is the minimum cardinality of a global dominating set of G, and a global dominating set of minimum cardinality is called a $\gamma_g(G)$-set.

Keywords: Domination number, total, connected, independent, global.
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Bigroup of Operations

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By analogy of bilattices [1, 2, 3] we consider the concepts of a bisemigroup, a bimonoid, a De Morgan bisemigroup and a bigroup.

A bisemigroup is an algebra $Q(\cdot, \circ)$ equipped with two binary associative operations $\cdot$ and $\circ$. If both of these operations have an identity element, then the bisemigroup is called a bimonoid. A commutative bisemigroup is a bisemigroup in which both operations are commutative. A bisemilattice is a commutative bisemigroup in which both operations are idempotent. In any bisemilattice $Q(\cdot, \circ)$, the binary operations determine two partial orders $\leqslant_1$ and $\leqslant_2$. A bisemilattice is called a bilattice, if the partial orders $\leqslant_1$ and $\leqslant_2$ are lattice orders. Since every lattice order is characterized by two binary operations, every bilattice is a binary algebra with four operations and corresponding identities. A De Morgan bisemigroup is an algebra $Q(\cdot, \circ, ^-, 0, 1)$ such that $Q(\cdot, \circ)$ is a bimonoid with identity elements 0 (for operation $\cdot$), 1 (for the operation $\circ$) and such that the identities

$\overline{\overline{x}} = x,$
$\overline{x \cdot y} = \overline{x} \circ \overline{y},$
$\overline{x \circ y} = \overline{x} \cdot \overline{y},$
$x \circ 0 = 0 \circ x = 0,$
$x \cdot 1 = 1 \cdot x = 1$

hold. A De Morgan bisemigroup $Q(\cdot, \circ, ^-, 0, 1)$ is a De Morgan algebra if $Q(\cdot, \circ)$ is a distributive lattice .

Let Q be an arbitrary non-empty set, let $O_p^{(n)}Q$ be a set of all n-ary operations on Q, and:

$$O_pQ = \bigcup_n O_p^{(n)}Q;$$

A bimonoid $Q(\cdot, \circ)$ with identity elements 0 (for operation $\cdot$) and 1 (for operation $\circ$) is called a bigroup, if

$x \circ 0 = 0 \circ x = 0,$
$x \cdot 1 = 1 \cdot x = 1,$

and the following conditions are valid:

a) $Q \setminus \{1\}$ is a group with an identity element 0 under the multiplication $\cdot$;

b) $Q\backslash\{0\}$ is a group with an identity element 1 under the multiplication $\circ$;

A bigroup of order > 3 is called non-trivial.

The set $O_p^{(2)}Q$ of all binary operations on the set Q is a bimonoid under the following operations:

$$f \cdot g(x, y) = f(x, g(x, y)), \tag{1}$$

$$f \circ g(x, y) = f(g(x, y), y), \tag{2}$$

in which the identity elements are the identical operations δ_2^2 and δ_2^1,where $\delta_2^1(x, y) = x$, and $\delta_2^2(x, y) = y$ for all $x, y \in Q$. Any subset $S \subseteq O_p^{(2)}Q$ which is closed under these two operations is called a bisemigroup of operations (on the set Q). The bisemigroup of operations (on the set Q) is called a bimonoid of operations (on the set Q) if it contains the identical operations δ_2^1 and δ_2^2.

The bimonoid S of operations (on the set Q) is a bigroup, if both of the following conditions are valid:

c) $S \setminus \{\delta_2^1\}$ is a group with an identity element δ_2^2 under the multiplication (1) ;

d) $S \setminus \{\delta_2^2\}$ is a group with an identity element δ_2^1 under the multiplication (2) ;

Such bigroup is called a bigroup of operations (on the set Q).

We characterize bigroups of operations in the category of second order algebras introduced in [4].

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On Characterization of Belousov Quasigroups

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The quasigroup $Q(\circ)$ is called a Belousov quasigroup,if the identities

$$x \circ (x \circ y) = y \circ x,$$

$$(x \circ y) \circ y = x,$$

$$x \circ (y \circ x) = (y \circ x) \circ y$$

are valid. A non-trivial Belousov quasigroup is not a Stein quasigroup and not commutative.

The set $O_p^{(2)}Q$ of all binary operations on the set Q is a monoid under the following operations:

$$f \cdot g(x, y) = f(x, g(x, y)), \tag{1}$$

$$f \circ g(x, y) = f(g(x, y), y). \tag{2}$$

Theorem 1. *If $Q(A)$ is a non-trivial Belousov quasigroup, then it is idempotent and $A \cdot A = A^*$, $A \cdot A^* = A \circ A^*$, $A \circ A = \delta_2^1$, $A^* \cdot A^* = \delta_2^2, A^* \circ A^* = A$. So if $Q(A)$ is a non-trivial Belousov quasigroup, then the set $\{\delta_2^1, \delta_2^2, A, A^*, A \cdot A^* = A \circ A^*\}$ is a bigroup of operations (on the set Q), where $A^*(x,y) = A(y,x)$ for every $x, y \in Q$.*

Theorem 2. *In every Belousov quasigroup $Q(\circ)$ the identities $(x \circ y) \circ (y \circ x) = y, (x \circ y) \circ (x \circ (y \circ x)) = y \circ x, (y \circ x) \circ (x \circ (y \circ x)) = x \circ y$ are valid. In a non-trivial Belousov quasigroup $Q(\circ)$, for any $a \neq b$ in Q the set $\{a, b, a \circ b, b \circ a, a \circ (b \circ a)\}$ is a five-element subquasigroup, which is isomorphic to the five-element quasigroup with the following multiplication table:*

	0	1	2	3	4
0	0	2	4	1	3
1	4	1	3	0	2
2	3	0	2	4	1
3	2	4	1	3	0
4	1	3	0	2	4

If we take such subquasigroups as blocks, we obtain a block design on the set Q.

It follows from the Theorem 2 that the non-trivial Belousov quasigroup has at least five elements. The variety of Belousov quasigroups is called a Belousov variety,which is a subvariety of the Mikado variety ([1]). Hence, the Belousov variety has a solvable word problem and is congruence-permutable. Every Belousov quasigroup of prime order is a simple algebra.

The applications of similar quasigroups in cellular automata see in [2].

To solution of the following problem is open.

To which loops are Belousov quasigroups isotopic?

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Stochastic tomography of convex bodies

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Let $\mathbf{R}^n$ $(n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset \mathbf{R}^n$ be a bounded convex body. Random k-flats in $\mathbf{R}^n$, $1 \leq k \leq n-1$ generate cross sections of random size in convex body $\mathbf{D}$. As $\mathbf{D}$ is a convex body, then obviously intersections of k-flats with $\mathbf{D}$ are always connected subsets of $\mathbf{R}^n$ for every $k \in \{1, \ldots, n-1\}$. It is natural to require that the corresponding distribution of random size of cross sections to be invariant with respect to the group of all Euclidean motions in $\mathbf{R}^n$. The determination of the distribution of size of cross sections has a long tradition of application to collections of bounded convex bodies forming structures in metal and crystallography. However, calculations of geometrical characteristics of random cross sections is often a difficult task. In a special case $k = 1$ we call the corresponding distribution function as the chord length distribution function. For $n = 2$ the list of known results was expanded after 2005 when N. G. Aharonyan and V. K. Ohanyan obtained the explicit formula of the chord length distribution function for a regular pentagon (see [1]).

Proposition 1. . *Let* $\mathbf{D}$ *be a convex planar polygon which has* m *pairs of parallel sides* $(a_{i_1}, a_{j_1}),\ldots,(a_{i_m}, a_{j_m})$. *The distances of the parallel lines which carry these segments are* $d_1,\ldots,d_m$, *respectively, and* $\pi a_{i_k} \cap \pi a_{j_k}$ *denotes the length of the intersection of the orthogonal projections of both segments onto one of the carrying lines,* $k = 1, \ldots, m$. *Then for* $k \in \{1, \ldots, m\}$ *for which* $\pi a_{i_k} \cap \pi a_{j_k} > 0$, *the chord length density function has a discontinuity at* d_k, *and the limit from above at* d_k *is infinite.*

A computer program is created which gives values of a chord length distribution function in the case of a regular n-gon for every natural $n \geq 3$.

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Minimizing the sequence of transitions of Petri nets using the reachable tree

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Abstract. This work is dedicated to minimizing the sequence of transitions of Petri nets using the reachable tree. The reachable tree is retrieved, corresponding to Petri Nets. Then the infinite reachable tree is replaced with "finite" tree, by introducing an item, which replaces the idea of an infinite. There is algorithm description of the minimal sequence of possible transitions. The designed algorithm gets the shortest possible sequence for the net advance state, which brings the mentioned net state into covering state.

There is theorem, which states that through the describing algorithm, the number of transitions in covering state is in minimal.

Keywords: Petri Nets, reachable trees, transition, position, covering condition.

Construction of discrete systems models need system components, with its operations in the abstract, such as, the program operator action, trigger transition from one state to another, interruptions in the operating system, machine or conveyer action, project phase completion etc. In general, the same system can operate differently in different conditions, bringing a multitude of processes, which means operating not deterministically. The real system operates in certain time, cases occur in certain periods and last for certain time. In synchronic models of discrete systems, the events are clearly associated with certain moments or pauses, during which all the components make simultaneous change in the system state, which is interpreted as a change in the system state. State conditions change successively. Alongside, these large systems, modeling approach has several drawbacks.

- First of all, the system must take into account all the components of its overall condition of each change, so that model appears formidable.
- Second, in such an approach, information are disappeared between casual links in systems.
- Thirdly, the so-called asynchronous systems may occur uncertain events at intervals of time.

The above-mentioned types of models, including Petri Nets are called asynchronous. Link replacements in time, with causal relationships, give chance to more clearly describe the structural features of the system.

Therefore, it is natural that many systems are suitable as discrete structures, consisting of two elements: the type of events and terms. The cases and terms in Petri Nets are disjoint sets with each other, respectively, called transitions and positions sets. Transitions are depicted in a graphical representation of Petri Nets (vertical lines), and places, with circles [1-3].

Conclusion.

The above mentioned studies and the proved theorem brings out several important features of Petri Nets in optimization perspective, according which, if Petri Nets are used in technical devices, then the idea of succession transition passages brings resources and saves time[4-6].

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Mathematical tomography

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Tomography and and its early history. Computerized tomography is one of the most impressive scientific achievements of the XXth century. It had a revolutionizing impact on the whole contemporary medical science and now it is difficult to imagine a serious medical hospital without a computerized tomograph. For its construction the physicists Allan Cormac and Godfrey Haunsfield were awarded in 1979 by the Nobel Prize in physiology and medical sciences.

Let us recall what is the computerized tomography. We all have passed through the Roentgen or X-ray diagnostics (called otherwise fluorography), say, of our breast. You are standing between the two vertical plates one of which is the source of X-rays while another one is the detector. As a result of such diagnostics you will obtain the two-dimensional projection of your breast on the detector plate, in other words its photo taken in Roentgen rays. If we can get the similar projections in different space directions then it would be possible to reconstruct with some accuracy the inner structure of your body. Such a reconstruction method was well known from the first half of XXth century and got the name of "tomography" (in Greek "$\tau o \mu o \sigma$" means "section"). However, this method could be realized in full scale only after the arrival of computer era. (Precisely by this reason the new reconstruction method is called the "computerized tomography".)

To take the computerized tomogram the patient is placed into a toroidal camera, surrounding the investigated part of his body. This camera contains both the sources and detectors of X-rays. The data obtained in the process of tomography characterize the decreasing of X-rays along a big set of straight lines piercing the investigated body in different directions lying in the plane of tomograph. If you want to get a 3-dimensional picture, you should move the body with respect to the camera thus reconstructing the body in different planes.

From mathematical point of view the reconstruction of one plane section of the body reduces to the reconstruction of a function on the plane from its integrals along all possible straight lines. This classical problem was solved by Johann Radon in 1917. Let us describe the Radon's solution in more detail. For that we consider a transform, called now the

Radon transform which associates with a function on the plane its integrals along all straight lines. Since the set of lines on the plane depends of two real parameters (the distance from the line to the origin and the angle between the line and real axis) we can pose the problem of existence of an inverse transform. Such inversion was found by Radon and it is given by an integral transform with the kernel determined (in modern terms) by some generalized function or distribution (later on we shall consider this Radon's formula in more detail).

For the sake of historical justice we should point out that before Radon another classical reconstruction problem was posed and solved by Paul Funk. Namely, suppose that we have an even function defined on the sphere in 3-dimensional space. By even function we mean a function which takes the same values in antipodal points of the sphere. The problem solved by Funk is formulated like this: is it possible to reconstruct such a function from its integrals along big circles (i.e. along equators of the sphere)? This problem was posed by Herman Minkowski and solved by him in principle with the help of decompositions into spherical functions (Minkowski's solution was published posthumously in the volume of his selected papers in 1911). Later on in 1913 Paul Funk has found a more elegant solution of this problem using the Abel's integral transform. We note in passing that we cannot get rid of the evenness condition of the original function since in the case of an arbitrary function on the sphere the Funk formula will not reproduce the original function. It gives the function which values in the antipodal points of the sphere are equal to the half sum of the values of the original function in these points (in particular, in the case of an odd function taking the opposite values in the antipodal points the Funk formula will produce the identical zero).

Mathematical meaning of Radon's formula.

Several words on the sense of Radon's formula from the point of view of the theory of distributions. It is based on the formula of decomposition of the delta-function in "plane waves". We recall that the plane wave in the n-dimensional Euclidean space is determined by a hyperplane with the normal ω so that its wave front at any given moment is given by the equation $(\omega, x) = \text{const}$. The formula of decomposition of the n-dimensional delta-function $\delta(x)$ in plane waves depends on the parity of the number n and looks differently in even-dimensional and odd-dimensional vector spaces. For odd n it is written as

$$\delta(x) = c_n \int_{S^{n-1}} \delta^{(n-1)}(\omega, x) d\omega \qquad (1)$$

where c_n is an explicit constant, depending only on the dimension of the space, $\delta^{(n-1)}(p)$ is the derivative of $(n-1)$th order of the delta-function

$\delta(p)$, taken for $p = (\omega, x)$, S^{n-1} is the unit sphere, and $d\omega$ is its area element. For even n (in particular, in the case $n = 2$ we are interested in) the decomposition formula has another form

$$\delta(x) = d_n \int_{S^{n-1}} \frac{d\omega}{(\omega, x)^n} \tag{2}$$

where d_n is again some constant, depending only on the dimension of the space. The kernel in this formula coincides with the regularization of the distribution p^λ, obtained by the analytic continuation of this function with respect to the complex parameter λ from the domain $\mathrm{Re}\,\lambda \geq 0$, where it is correctly defined, into the domain $\mathrm{Re}\,\lambda < 0$. The kernel in the formula (2) corresponds to the value of this distribution for $p = (\omega, x)$, $\lambda = -n$. Note that the formula (1) is of local character opposite to the formula (2).

The Radon inversion formula is obtained by the convolution of the the given function (equal to the Radon transform of the original function) with formula (2) .

X-ray tomograph.

Let us return to the X-ray tomograph. It seems that the Radon inversion formula resolves completely the problem of reconstruction of a given function from its integrals along straight lines. And it is indeed so on the level of theoretical mathematics. We have only to approximate the integral in Radon's formula by the discrete integral sum and everything is done: with the help of such approximation for any given $\varepsilon > 0$ we can reconstruct the original function from a discrete set of straight line with the given precision ε.

However the situation in practice is not that simple. As we have seen above, the kernel in the Radon inversion formula is a distribution which means that the integral of a given function with this kernel converges only in a weak sense. In practice it means that after the discretization such a convergence may be lost. By this reason the Cormack algorithm, realized in modern tomographs, is based on a completely different inversion formula. Namely, to get this formula we first apply to a given function the one-dimensional Fourier transform along the direction, normal to the considered straight line being the argument of our function. As a result, we obtain the two-dimensional Fourier transform which may be inversed by the two-dimensional inverse Fourier transform. This transform was extensively studied in the second half of XXth century. It can be realized using well-established modern computer algorithms. However, one should not think that we got rid of all our difficulties. The inversion of the Fourier transform, as also the majority of inverse problems, is an ill-posed problems. And this results in many serious difficulties in the process of its realization on computers. We shall not speak about these problems here but it is necessary to have them in mind. We note only that the high cost of modern computerized tomographs is explained mostly not by the technological (hardwear) problems but rather by the incorporated non-trivial algorithms (softwear) which constitute the main commercial secret.

The modern computerized tomographs can work in the real-time regime and are characterized by the high precision discovering the slightest differences in the density of the studied tissues (of order of fractions of a percent). Such a precision is sufficient to distinguish even the small cancer tissues at the beginning stage of their development.

Curvilinear Radon transform and its applications.

However, there are practical situations when the application of computerized tomography is not recommended because of the hardness of X-ray radiation (for example, it cannot be used to analyze the state of a pregnant woman). In such situations physicians prefer to use instead of X-rays less hard ultrasound rays. In contrast with X-rays, spreading along straight lines, the ultrasound waves propagate along the curved trajectories.

The corresponding mathematical problem can be formulated in the following way. Suppose that we have in the unit disk in the plane a conformally flat metric of the form

$$ds^2 = f(x, y)(dx^2 + dy^2).$$

We can assume that this metric is sufficiently good in the sense that any two points on the unit circle are connected by a unique geodesic of our metric. (One can suppose, for example, that the considered metric is close to a flat one which guarantees the fulfillment of the above condition.) Assume that we know the lengths $\ell(\varphi, \psi)$ of all geodesics, connecting the points on the circle with angle coordinates φ and ψ. We would like to reconstruct the original metric, i.e. the function $f(x, y)$, from these data. Since both functions depend on two real parameters, we can expect that this problem in principle can be solved. Indeed, it is proved that it is well-posed and correct which means that it has a unique solution and this solution depends continuously on the initial data. However, we do not know any explicit formula for such a solution, similar to that of Radon.

Generalizing this problem, we can consider the curvilinear Radon transform which associates with a given function in the disk its integrals over geodesics of a given metric and try to find an inversion formula for such transform. It is clear that the problem of ultrasound reconstruction is a particular case of this inversion problem. Unfortunately, this inversion problem still remains unsolved despite many efforts of mathematicians. By this reason ultrasound tomographs use in the curvilinear case Radon's formula for the straight line case which implies, of course, the low precision of ultrasound tomographs compared to their X-ray prototypes. The improvement of precision of ultrasound tomographs depends directly on the progress in the solution of the mathematical problem, formulated above.

The curvilinear Radon inversion problem is important not only in the medicine but also, for example, in geology where it arises when using the seismic methods of exploration of minerals. In the simplified form these methods work as follows. Assume that somewhere under the surface of the earth (mathematically, in the lower halfplane) we have an object (mineral) which we would like to localize. For that we arrange in different points of the surface of the earth (mathematically, the real line) a series of microexplosions. The neighboring seismic stations fix the seismic waves coming from these explosions. The velocity of propagation of these waves allows to calculate the lengths of geodesics of the metric characterizing the object we are interested in. It remains only to solve the inversion problem for the arising curvilinear Radon transform in order to reconstruct the metric and so localize the desired mineral. Hence, in this case the progress in developing of seismic methods also depends on the solution of the inversion problem for the curvilinear Radon transform.

There are still many unsolved mathematical problems related to tomography but we are sure that any progress in their solution will find important practical applications.