Կազմակերպչական կոմիտե՝

Ա. Բադալյան, Լ. Գևորգյան, Ա. Թասլաքյան, Գ. Խաչատրյան,
Խ. Խաչատրյան, Ռ. Համբարձումյան, Մ. Հարությունյան,
Ս. Հարությունյան, Տ. Հարությունյան (նախագահ), Հ. Ղազարյան,
Ա. Մարգարյան, Հ. Մկոյան, Յու. Մովսիսյան, Ս. Ռաֆայելյան,
Ա. Սահակյան, Վ. Սահակյան

Organizing committee

R. Ambartzumian, A. Badalyan, L. Gevorgyan, H. Ghazaryan,
S. Haroutunian, M. Harutyunyan, T. Harutyunyan (chairman),
G. Khachatryan, Kh. Khachatryan, A. Margaryan, H. Mkoyan,
Yu. Movsisyan, S. Rafayelyan, A. Sahakyan, V. Sahakyan,
A. Taslakyan

http://www.badalyan100.com/

1915-2004

Հայկ Վաղարշակի Բադալյանը ծնվել է 1915 թ. հունիսի 22-ին Կոտայքի մարզի Նոր Գյուղում: 1927-28 թթ. ընտանիքը փեղափոխվում է Երևան, որտեղ սովորում և ավարտում է դպրոցը, էլեկտրատեխնիկական ուսումնարանը եւ 1936-ին ընդունվում է Երևանի պետական համալսարանի ֆիզիկա-մաթեմատիկական ֆակուլտետ: Դիպլոմային աշխատանքը, ԵՊՀ ավարտական քննությունները եւ Հայրենական մեծ պատերազմի սկիզբը կարծես միաժամանակ են տեղի ունենում 1941 թվականին: Արիավիրքի սկզբից Հ.Վ. Բաղալյանը, ինչպես կամավոր, համալրում է Սովետական զինուժը եւ ծառայում է բանակի թիկունքային ստորաբաժանումներում մինչ 1943 թ. վերջը: 1944-47 թթ. շարունակում է ուսումնառությունը Մոսկվայի պետական համալսարանի ասպիրանտուրայում, որտեղ էլ 1947 թ. հաջողությամբ պաշտպանում է «Ընդհանրացված ֆակտորիալ շարքեր» թեմայով թեկնածուական թեզը:

Վերադառնալով հայրենիք, Հ.Վ. Բաղալյանը աշխատանքի է անցնում Երևանի Ա.Ա. Ժդանովի անվան Ռուսական մանկավարժական ինստիտուտում՝ որպես մաթեմատիկայի ամբիոնի վարիչ, համատեղելով մանկավարժությունը գիտահետազոտական գործունեությանը՝ ՀԽՍՀ ԳԱ մաթեմատիկայի ինստիտուտում:

«Անալիտիկ եւ քվազիանալիտիկ ֆունկցիաների տեսության որոշ հարցեր» թեմայով դոկտորական թեզը, որը Հ.Վ. Բաղալյանը պաշտպանում է 1956 թվականին Մոսկվայի պետական համալսարանում, բարձր է գնահատվում ճանաչված գիտնականների կողմից: Ահա մի քանի արտահայտություններ. պրոֆեսորներ Ս.Ն. Մերգելյան և Ա.Ի.Մարկուշևիչ՝ «..Հ.Վ. Բաղալյանը ստացել է Կարլեմանի (*T. Carleman*) պրոբլեմի էֆեկտիվ լուծում միանգամայն նոր մեթոդների շնորհիվ..»: Պրոֆեսոր Ա.Օ. Գելֆոնդ՝ «Հ.Վ. Բաղալյանը մշակել է քվազիանալիտիկ ֆունցիաների մի այնպիսի ապարատ, որը հնարավոր է դարձնում այդ տիպի ֆունկցիաների ռեգուլյար հետազոտումները»: Պրոֆեսոր Ի.Պ. Նատանսոն՝ «..Հ.Վ. Բաղալյանի քվազիանալիտիկ ֆունցիաների տեսության հետազոտությունները ինչպես և այդ տաղանդավոր գիտնականի այլ աշխատանքները ներկայացնում են բացառիկ հետաքրքրություն..»:

Ի հետևանք 1957 թ. Ռուսական մանկավարժական ինստիտուտի ֆիզմաթ ֆակուլտետի միացումն ԵրՊետ համալսարանին, Հ.Վ. Բաղալյանի հետագա գիտամանկավարժական գործունեությունը կապվում է ԵՊՀ ֆիզ-մաթ (ներկայումս մաթեմատիկայի) ֆակուլտետի հետ: 1960 թ. նրան շնորհվում է պրոֆեսորի կոչում: 1978-87 թթ. Հ.Վ. Բաղալյանը ղեկավարում է Մաթեմատիկական անալիզի ամբիոնը, զուգակցելով այդ աշխատանքը «Մաթեմատիկա» միջբուհական տեղեկագրի գլխավոր խմբագրի պաշտոնի հետ: Այդ տարիներին ընդլայնվում և առավել ընդգրկուն է դառնում պրոֆեսորի գիտական նախասիրությունների շրջանակները՝ անվերջ դիֆերենցելի ֆունկցիաներ, քվազիանալիտիկ ֆունկցիաների դասեր, ասիմպտոտիկ վերլուծություններ, օրթոգոնալ համակարգեր, Ռիմանների կորիզով ինտեգրալային ձևափոխություններ, անկյունային տիրույթներում Գելֆոնդ-Շիլովյան տիպի խնդիրների միակություն, ամբողջ եւ քվազի-ամբողջ ֆունկցիաների վերլուծությունների լակունարության և բիօրթոգոնալ համակարգերի հարցեր:

Քվազիանալիտիկ դասերի տեսությունում, նոր մոտեցումների և յուրահատուկ տեխնիկայի շնորհիվ, Հ.Վ. Բադալյանին հաջողվում է առաջադրել և լուծել մի շարք հիմնախնդիրներ: Այդ արդյունքները հանդիսանում են հայտնի մաթեմատիկոսներ Վաթսոնի (*G.N. Watson*), Բորելի (*E. Borel*), Մանդելբրոյտի (*S. Mandelbrojt*), Գելֆոնդի (*A.O. Gelfond*) և այլոց աշխատանքների շարունակությունն: Ակնառու գիտնականը կարևոր արդյունքներ է ստացել նաև անվերջ դիֆերենցելի ֆունկցիաների տեսության, Մյունցի (*Ch. Muntz*) սիստեմի բազմանդամներով մոտարկման, Թեյլորյան (*B. Teylor*) տիպի բանաձևերի կառուցման, փաթեթային տիպի ինտեգրալային ձևափոխությունների, օրթոգոնալ համակարգերի կառուցման եւ շատ այլ բնագավառներում:

Հ.Վ.Բադալյանի մոտ 80 գիտական աշխատությունները հրապարակվել են հայկական, նախկին ՍՍՀՄ կենտրոնական մաթեմատիկական ամսագրերում, Եվրոպայի եւ ԱՄՆ-ի առաջատար մաթեմատիկական հանդեսներում: Դրանցում շարադրված առավել արժեքավոր արդյունքներն, այդ թվում եւ վերն արդեն նշված անվանական պրոբլեմները, ամփոփվել են «Նաուկա» հրատարակչության կողմից 1990թ. հրապարակված «Քվազիաստիճանային շարք և ֆունկցիաների քվազիանալիտիկ դասեր» մենագրությունում:

Նշված մենագրությունը 2002-ին հրապարակվել է ԱՄՆ-ում Ամերիկյան Մաթեմատիկական Միության (*American Mathematical Society*) կողմից «*Quasipower Series and Quasianalytic Classes of Functions*» վերնագրով: Ի տարբերություն մոսկովյան հրապարակության, անգլերեն տարբերակը ընդգրկել է հեղինակի կողմից վերջին տարիներին ստացված լրացուցիչ նյութեր, որոնք վերաբերվում են անվերջ դիֆերենցելի ֆունկցիաների դասակարգման պրոբլեմին, բացահայտում են քվազիաստիճանային շարքերի բացարձակ զուգամիտության պայմանները և անալիտիկ ֆունկցիաների ոչ շրջանային տիրույթում քվազիաստիճանային շարքերով ներկայացման հնարավորությունը:

Ըստ Ամերիկյան Մաթեմատիկական Միության խմբագիր Ջեյմս Ստաշեֆֆի, «Հ.Վ. Բադալյանի մենագրության մեջ ներկայացված քվազիաստիճանային շարքերի վերաբերյալ տեխնիկան հնարավորություն է տալիս ընդլայնել արդեն հայտնի արդյունքները և լուսաբանել նրանց բնույթը յուրահատուկ կերպով: Բոլոր նշված ուսումնասիրություններն աչքի են ընկնում ոչ միայն խնդիրների ինքնատիպ դրվածքով, այլև լուծման նոր և, որպես կանոն էֆեկտիվ ու վերջնական արդյունքներով: Հեղինակի մեթոդները ընձեռում են լայնածավալ ստեղծագործական հնարավորություններ երիտասարդ մաթեմատիկոսների համար»:

Ավելացնենք, որ գրքի հրապարակումից հետո պրոֆեսոր Հ.Վ. Բադալյանը ընտրվեց ԱՄՆ մաթեմատիկական միության իսկական անդամ, իսկ մենագրությունը ներկայումս օգտագործվում է որպես սեմինարների ձեռնարկ Կալիֆորնիայի Տեխնոլոգիական Ինստիտուտում:

Անգնահատելի է Հ.Վ. Բադալյանի դերը Հայաստանի մաթեմատիկական դպրոցի ստեղծման և երիտասարդ մաթեմատիկոս կադրերի պատրաստման գործում: Նրա գիտական ղեկավարությամբ ներկայացվել և պաշտպանվել են թեկնածուական թեզեր:

Բավականին առաջացած տարիքում պրոֆեսոր Հ.Վ. Բաղալյանը հավա–տարիմ մնալով ստեղծագործող գիտնականի իր բնույթին շարունակում էր ստեղծագործել և հրապարակել նոր աշխատություններ:

Բովանդակություն/Content

Ternary hyperidentities of associativity defined by the equality $((x, y, z), u, v) = (x, y, (z, u, v))$

L.R. Abrahamyan

Artsakh State University, Artsakh
E-mail: *liana_abrahamyan@mail.ru*

Beginning at the 1960s, the following formula from a second-order language with specialized quantifiers have been studied in various domains of algebra and its applications:

$$\forall X_1, \ldots, X_m \forall x_1, \ldots, x_n (W_1 = W_2), \tag{1}$$

where w_1, w_2 are terms (words) in the functional variables $X_1, ..., X_m$ and in the object variables $x_1, ..., x_n$. This formula is called $\forall(\forall)$-identity or hyperidentity. For simplicity the hyperidentity is written without the quantifier prefix, i.e. as an equality: $w_1 = w_2$. The number m is called functional rank and the number n is called object rank of the given hyperidentity. A hyperidentity is said to hold (or satisfied) in an algebra $(Q; U)$ if the equality $w_1 = w_2$ is valid when every object variable and every functional variable in it is replaced respectively by any arbitrary element of Q and any operation of the corresponding arity from U (it is assumed that such replacement is possible).

Theorem 1. *If in a nontrivial invertible algebra a nontrivial hyperidentity of associativity is satisfied, which is defined by the equality:*

$$((x, y, z), u, v) = (x, y, (z, u, v)),$$

then every functional variable is repeated in it at least twice. Therefore,each of such hyperidentities can only be of functional rank 2 and one of the following types:

$$1. X(Y(x, y, z), u, v)) = Y(x, y, X(z, u, v)),$$

$$2. X(X(x, y, z), u, v)) = Y(x, y, Y(z, u, v)),$$

$$3. X(Y(x, y, z), u, v)) = X(x, y, Y(z, u, v)).$$

Then we obtain characterizations of the ternary invertible algebras with these hyperidentities.

References

[1] A. I. Malcev, *Some problems in the theory of classes of models*, Proceedings of IV All-Union Mathematical Congress, Leningrad, 1, Publishing House of the USSR Academy of Sciences, Leningrad, 169-198(1963).

[2] A. Church, *Introduction to mathematical logic*, vol. I, Princeton University Press, Princeton, 1956.

[3] Yu.M. Movsisyan, *Introduction to the theory of algebras with hyperidentities*, Yerevan State University Press, Yerevan, 1986. (Russian)

[4] Yu.M.Movsisyan, *Hyperidentities and hypervarieties in algebras*, Yerevan State University Press, Yerevan, 1990 (Russian).

[5] Yu.M. Movsisyan, Hyperidentities in algebras and varieties, *Uspekhi Matematicheskikh Nauk*, 53 (1998), pp. 61–114. English translation in Russian Mathematical Surveys 53(1998), pp. 57–108.

[6] Yu.M. Movsisyan, Hyperidentities and hypervarieties, *Scientiae Mathematicae Japonicae*, 54(3), (2001), 595–640.

Density function of the distance between two random points in a convex set

N.G. Aharonyan

Yerevan State University, Armenia
E-mail: *narine78@ysu.am*

Let $\mathbf{D}$ be a bounded, convex domain in the Euclidean plane, with the area $\|\mathbf{D}\|$ and the perimeter $|\partial\mathbf{D}|$. Let P_1 and P_2 be two points chosen at random, independently and with uniform distribution in $\mathbf{D}$. We are going to find the density function of the distance $\rho(P_1, P_2)$ between P_1 and P_2.

Firstly, we find the distribution function $F_\rho(x)$ of $\rho(P_1, P_2)$. By definition,

$$F_\rho(x) = \frac{1}{\|\mathbf{D}\|^2} \iint\limits_{\{(P_1,P_2):\rho(P_1,P_2)\leq x\}} dP_1\, dP_2, \tag{1}$$

where dP_i, $i = 1, 2$ is the Lebesgue measure in the plane $\mathbf{R}^2$.

From the expression of the area element in polar coordinates we have

$$dP_1\, dP_2 = r\, dP_1\, dr\, d\varphi, \tag{2}$$

where φ is the angle between the line through the points P_1, P_2 and the reference direction in the plane. If we leave r fixed, then $dP_1\, d\varphi$ is the kinematic density for the segment P_1P_2 of length r.

Using (2) we can rewrite (1) in the form:

$$F_\rho(x) = \frac{1}{\|\mathbf{D}\|^2} \int_0^x r\, K(\mathbf{D}, r)\, dr, \tag{3}$$

where $K(\mathbf{D}, r)$ is the kinematic measure of all oriented segments of length r that lie inside $\mathbf{D}$. Therefore, we obtain a relationship between the density function $f_\rho(x)$ of $\rho(P_1, P_2)$ and the kinematic measure $K(\mathbf{D}, r)$:

$$f_\rho(x) = \frac{x\, K(\mathbf{D}, x)}{\|\mathbf{D}\|^2}. \tag{4}$$

Note that we can calculate the kinematic measure of all unoriented segments that lie inside $\mathbf{D}$ and then the result multiplied by 2.

As it is well-known (see [1], [2]), the solution of the problem on finding the kinematic measure $K(\mathbf{D}, r)$ of segments with constant length r, contained in $\mathbf{D}$, is not simple and essentially depends on the form of $\mathbf{D}$.

Explicit expressions for $K(\mathbf{D}, r)$ are known only in two cases (see [1], [6]) in a disc and a rectangle.

In the paper [3], a formula for the kinematic measure $K(\mathbf{D}, r)$ of sets of segments with constant length r entirely contained in $\mathbf{D}$ is obtained.

$$f_\rho(x) = \frac{1}{\|\mathbf{D}\|^2} \left[2\pi\, x\, \|\mathbf{D}\| - 2\, x^2\, |\partial \mathbf{D}| + 2\, x\, |\partial \mathbf{D}| \int_0^x F_{\mathbf{D}}(u)\, du \right], \qquad (5)$$

where $F_{\mathbf{D}}(\cdot)$ is the chord length distribution function for the domain $\mathbf{D}$. The obtained formula permits to calculate the mentioned kinematic measure $K(\mathbf{D}, r)$ by means of the chord length distribution function of $\mathbf{D}$. Therefore if we know the explicit form of the chord length distribution function for a domain, using (5) we can calculate density function $f_\rho(x)$ of the distance between two random points in $\mathbf{D}$. In [5] the explicit form of the chord length distribution function is given for any regular polygon. Consequently, density $f_\rho(x)$ can be calculated for any regular polygon by applying the result of [5] (see also [4]) and formula (5).

References

[1] Luis A. Santalo, Integral geometry and geometric probability. *(Addision-Wesley, Reading, MA,)*, 2004.

[2] R. Schneider and W. Weil. Stochastic and Integral Geometry, *Springer-Verlag, Berlin Heidelberg*, 2008.

[3] N. G. Aharonyan, and V. K. Ohanyan, Kinematic measure of the segments lie in a domain, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 46 (5): 3 – 14, 2011.

[4] N. G. Aharonyan and V. K. Ohanyan, Chord length distribution functions for polygons, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 40 (4): 43 – 56, 2005.

[5] H. S. Harutyunyan and V. K. Ohanyan, Chord length distribution function for regular polygons, *Advances in Applied Probability*, 41: 358 – 366, 2009.

[6] N. G. Aharonyan, H. S. Harutyunyan and V. K. Ohanyan, Random copy of a segment within a convex domain, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 45 (6): 348 – 356, 2010.

On properties of the spectral data of Sturm-Liouville boundary-value problems

Yu. Ashrafyan

Yerevan State University, Armenia
E-mail: *yuriashrafyan@ysu.am*

Introduction. We consider the Sturm-Liouville operator

$$\ell := -\frac{d^2}{dx^2} + q(x), \tag{1}$$

with boundary conditions

$$y'(0) + y(0)\cot\alpha = 0, \tag{2}$$

$$y'(\pi) + y(\pi)\cot\beta = 0, \tag{3}$$

where $\alpha, \beta \in (0, \pi)$ and q is a real-valued functions which is integrable on $[0, \pi]$ (we write $q \in L^1_{\mathbb{R}}[0, \pi]$). By $L(q, \alpha, \beta)$ we denote the self-adjoint operator, generated by problem (1)-(3). It is known, that under these conditions the spectra of the operator $L(q, \alpha, \beta)$ is discrete and consists of real, simple eigenvalues, which we denote by $\mu_n = \mu_n(q, \alpha, \beta)$, $n = 0, 1, 2, \dots$, emphasizing the dependence of μ_n on q, α and β.

Let $\varphi(x, \mu, \alpha, q)$ and $\psi(x, \mu, \beta, q)$ be the solutions of the operator (1), which satisfy the initial conditions

$$\varphi(0, \mu, \alpha, q) = 1, \quad \varphi'(0, \mu, \alpha, q) = -\cot\alpha,$$
$$\psi(\pi, \mu, \beta, q) = 1, \quad \psi'(\pi, \mu, \beta, q) = -\cot\beta,$$

respectively.

It is easy to see that the functions $\varphi_n(x) := \varphi(x, \mu_n, \alpha, q)$ and $\psi_n(x) := \psi(x, \mu_n, \beta, q)$, $n = 0, 1, 2, \dots$, are the eigenfunctions, corresponding to the eigenvalue μ_n.

The squares of the L^2-norm of these eigenfunctions divided by $\sin^2\alpha$ and $\sin^2\beta$ respectively:

$$a_n = a_n(q, \alpha, \beta) := \int_0^\pi \frac{|\varphi_n(x)|^2}{\sin^2\alpha} dx, \quad n = 0, 1, 2, \dots,$$
$$b_n = b_n(q, \alpha, \beta) := \int_0^\pi \frac{|\psi_n(x)|^2}{\sin^2\beta} dx, \quad n = 0, 1, 2, \dots$$

are called norming constants.

The inverse problem by "spectral function" is the reconstruction of the problem $L(q,\alpha,\beta)$ from the spectra $\{\mu_n\}_{n=0}^{\infty}$ and the norming constants $\{a_n\}_{n=0}^{\infty}$ (or $\{b_n\}_{n=0}^{\infty}$). The two sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ together will be called the spectral data. Necessary and sufficient conditions for two sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ to be the spectral data for some Sturm-Liouville problem are well known. In this thesis we state the question:

what kind of sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ should be to become the spectral data for problem $L(q,\alpha,\beta)$ with $q \in L^2_{\mathbb{R}}[0,\pi]$ and in advance fixed α and β in $(0,\pi)$?

Our answer is in the following assertions.

Theorem 1. *Let $q \in L^2_{\mathbb{R}}[0,\pi]$ and $\alpha,\beta \in (0,\pi)$. Then for norming constants $a_n = a_n(q,\alpha,\beta)$ and $b_n = b_n(q,\alpha,\beta)$ satisfy*

$$\frac{1}{a_0} - \frac{1}{\pi} + \sum_{n=1}^{\infty}\left(\frac{1}{a_n} - \frac{2}{\pi}\right) = \cot\alpha, \tag{4}$$

$$\frac{1}{b_0} - \frac{1}{\pi} + \sum_{n=1}^{\infty}\left(\frac{1}{b_n} - \frac{2}{\pi}\right) = -\cot\beta. \tag{5}$$

Theorem 2. *For a real increasing sequence $\{\mu_n\}_{n=0}^{\infty}$ and a positive sequence $\{a_n\}_{n=0}^{\infty}$ to be spectral data for an operator $L(q,\alpha,\beta)$ with a $q \in L^2_{\mathbb{R}}[0,\pi]$ and fixed $\alpha,\beta \in (0,\pi)$ it is necessary and sufficient that the following relations hold:*

$$\sqrt{\mu_n} = n + \frac{\omega}{\pi n} + \frac{\omega_n}{n}, \quad \omega = const, \quad \{\omega_n\}_{n=0}^{\infty} \in l^2, \tag{6}$$

$$a_n = \frac{\pi}{2} + \frac{\kappa_n}{n}, \quad \{\kappa_n\}_{n=0}^{\infty} \in l^2, \tag{7}$$

$$\frac{1}{a_0} - \frac{1}{\pi} + \sum_{n=1}^{\infty}\left(\frac{1}{a_n} - \frac{2}{\pi}\right) = \cot\alpha, \tag{8}$$

$$\frac{a_0}{\pi^2 \cdot \left(\prod_{k=1}^{\infty} \frac{\mu_k - \mu_0}{k^2}\right)^2} - \frac{1}{\pi} +$$

$$+ \sum_{n=1}^{\infty}\left(\frac{a_n n^4}{\pi^2[\mu_0 - \mu_n]^2 \left(\prod_{k=1, k\neq n}^{\infty} \frac{\mu_k - \mu_n}{k^2}\right)^2} - \frac{2}{\pi}\right) = -\cot\beta. \tag{9}$$

References

[1] M. A. Naimark., Linear Differential Operators. *Nauka, Moscow*, 1969, (in Russian).

[2] E. L. Isaacson, E. Trubowitz. The Inverse Sturm-Liouville Problem, I. *Com. Pure and Appl. Math.*, Vol 36, pp. 767-783, 1983.

On weighted Dirichlet spaces in the ball

K. Avetisyan

Yerevan State University, Armenia
E-mail: *avetkaren@ysu.am*

The weighted Dirichlet spaces of holomorphic or harmonic functions on the complex plane are well studied, for example, the monograph [1]. We define and study some analogues of weighted Dirichlet spaces for monogenic functions with values in the reduced quaternions, see [2]. The scale of weighted Dirichlet spaces somewhat differs from that of classical ones. Some sharp inclusions are established for monogenic and harmonic Dirichlet spaces. Corresponding counter-examples are given.

References

[1] K. Zhu. Operator theory in function spaces. *Pure and Applied Mathematics.*, 136, Marcel Dekker, Inc., New York, 1990.

[2] J. Morais, K. Avetisyan and K. Gürlebeck. On Riesz systems of harmonic conjugates in $\mathbb{R}^3$. *Math. Meth. Appl. Sci.*, 36(12):1598–1614, 2013.

On index invariance of semi-elliptic operator

A.A. Darbinyan, A.G. Tumanyan

Russian-Armenian Slavonic University, Armenia
E-mail: *armankri@yahoo.com, ani.tumanyan92@gmail.com*

Introduction. This paper is devoted to research on Noethericity and the invariance of the index of semi-elliptic operator on the scale of anisotropic spaces.

Noethericity (see [1]) in smooth compact manifolds is proven for elliptic operators [2], formula is obtained for the index in topological terms [3]. In [2], as a consequence, the invariance of the index on the scale of Sobolev spaces defined on compact manifolds is obtained.
For semi-elliptic operators previously were obtained following main results. The class of semi-elliptic operators with constant coefficients in R^n is described [4], Noethericity is proven for one class of semi-elliptic operators with variable coefficients in weighted Sobolev spaces [5].
Let

$$P\left(x,D\right)=\sum_{(\alpha:\nu)\leq s}a_{\alpha}(x)D^{\alpha},$$

where $\alpha,\nu\in Z_{+}^{n},\nu\neq 0,\ (\alpha:\nu)=\frac{\alpha_1}{\nu_1}+...+\frac{\alpha_n}{\nu_n},\ s\in N,\ D^{\alpha}=D_1^{\alpha_1}...D_n^{\alpha_n},\ D_k=i\frac{\partial}{\partial x_k},x=(x_1,...,x_n)\in R^n,\ n\geq 2,\ a_{\alpha}(x)$ are infinitely differentiable and bounded together with all derivatives.
By $P_s(x,D)$ denote principal part of differential expression $P(x,D)$ and let $\sigma_s\left(x,\xi\right)$ be the symbol of principal part.
For $k\in R,\nu\in Z_+^n$, let $H_{\nu}^{k}(R^n)$ denote

$$H_{\nu}^{k}(R^n)\equiv\{u\in S^{'}:\|u\|_{k,\nu}(R^n)=(\textstyle\int_{R^n}|\hat{u}(\xi)|^2(1+|\xi|_{\nu})^{2k}d\xi)^{1/2}<\infty\},$$

where $|\xi|_{\nu}=(\sum_{i=1}^{n}|\xi_i|^{2\nu_i})^{1/2}$, $S^{'}$ is the space of generalized functions of slow growth, $\hat{u}$ is Fourier transform of u.
Let $\Omega\subset R^n$ be some domain. Denote by $\dot{H}_{\nu}^{k}\left(\Omega\right)$ completion of $C_0^{\infty}\left(\Omega\right)$ with the norm of $H_{\nu}^{k}\left(R^n\right)$.

Definition 1. *We say that $P\left(x,D\right)$ is semi-elliptic at $x=x_0$, if*

$$\sigma_s\left(x_0,\xi\right)\neq 0,\forall\xi\in R^n,|\xi|\neq 0.$$

Definition 2. *We say that $P\left(x,D\right)$ is uniformly semi-elliptic, if there exists a positive constant C, such that*

$$|\sigma_s(x,\xi)| \geq C\,|\xi|_\nu^s\,, \forall x \in R^n, \forall \xi \in R^n.$$

We will use following notations for $P(x,D)$:

$$\Delta(P) = \max_{(\alpha:\nu)=s} \sup_{x\in R^n} |a_\alpha(x) - a_\alpha(0)|\,, \delta = \inf_{|\xi|_\nu^s=1} |\sigma_s(0,\xi)|.$$

In this work following theorem on the sufficient condition for the invariance of the index and Noethericity preserving is proved:

Theorem 1. *Let $P(x,D) : H_\nu^{k+s}(R^n) \to H_\nu^k(R^n)$ be semi-elliptic operator at $x = 0$, k_0 is some fixed positive number and there exists a positive number ε_0, depending on k_0 HD δ, such that $\Delta(P) < \varepsilon_0$. Then for arbitrary k_1 HD k_2, such that $k_1, k_2 \in [0, k_0]$ and $k_1 < k_2$ following applies:*

if $P(x,D) : H_\nu^{k_1+s}(R^n) \to H_\nu^{k_1}(R^n)$ is Noetherian, then $P(x,D) : H_\nu^{k_2+s}(R^n) \to H_\nu^{k_2}(R^n)$ is also Noetherian and $ind_{k_1}(P) = ind_{k_2}(P)$.

For uniformly semi-elliptic operator the following is proved:

Theorem 2. *Let $P(x,D) : \dot{H}_\nu^{k+s}(\Omega) \to \dot{H}_\nu^k(\Omega)$ be uniformly semi-elliptic operator. If the operator $P(x,D) : \dot{H}_\nu^{k_1+s}(\Omega) \to \dot{H}_\nu^{k_1}(\Omega)$ is Noetherian for some value k_1, then $P(x,D) : \dot{H}_\nu^{k+s}(\Omega) \to \dot{H}_\nu^k(\Omega)$ is Noetherian for any k, and $ind_k(P)$ does not depend on k.*

References

[1] Kutateladze S. S. Fundamentals of Functional Analysis, *Kluwer Texts in the Mathematical Sciences*, vol. 12, Kluwer Academic Publishers Group, Dordrecht, 1996

[2] Agranovich M. S. Elliptic singular integro-differential operators, *Uspekhi Mat. Nauk*, 20:5(125) (1965), pp. 3-120

[3] Atiyah M. F., Singer I.M. The index of elliptic operators on compact manifolds, *Bull. Amer. Math. Soc.*, 1963, V. 69, pp. 422-433

[4] Darbinyan A. A., Tumanyan A.G. Necessary and sufficient condition of Noethericity for operators with constant coefficients, *Proceedings of Russian-Armenian (Slavonic) University (Physical, mathematical and natural sciences)*, 2014 #2 pp. 4-14, Yerevan, 2014

[5] Karapetyan G. A., Darbinyan A. A. Index of semi-elliptic operator in R^n, *Proceedings of the NAS Armenia: Mathematics* ,V. 42, #5, pp. 33-50, Yerevan, 2007

On cyclability of Hamiltonian digraphs

S.Kh. Darbinyan

Institute for Informatics and Automation Problems of NAS, Armenia
E-mail: *samdarbin@ipia.sci.am*

Terminology and notations not described below follows [1]. We consider finite digraphs without loops and multiple arcs. Every cycle and path is assumed simple and directed.

Let D (G) be a directed graph D (an undirected graph G) of order n and let S be a nonempty subset of D (G).The subset S is said to be cyclable in D (in G) if D (if G) contains a directed cycle (undirected cycle) through all the vertices of S. We say that a digraph D is S-strongly connected if for any pair x, y of distinct vertices of S there exists a path from x to y and a path from y to x in D. A Meyniel set M is a subset of vertices of D such that $d(x) + d(y) \geq 2n - 1$ for every pair of distinct vertices x, y in M which are nonadjacent in D.
There are many well-known conditions which guarantee the cyclability of a set of vertices in an undirected graph. Most of them can be seen as restrictions of Hamiltonian conditions to the considered set of vertices ([2],[3] and [4]). For general directed graphs (digraphs) there are not in literature as many conditions as for undirected graphs that guarantee the existence of a directed cycle with given properties (in particular, sufficient conditions for the existence of a Hamiltonian cycles in a digraphs). The more general and classical ones is Meyniel's theorem.
Theorem A (*Meyniel [6]*). *Let D be a strongly connected digraph of order $n \geq 2$ and $d(x) + d(y) \geq 2n - 1$ for all pairs of nonadjacent vertices in D. Then D is Hamiltonian.*

Recall that Meyniel's theorem is a common generalization of well-known classical theorems of Ghouila-Houri and Woodall (e.g., see [1]).

Sufficient conditions for cyclability in digraphs with the condition of Meyniel's theorem was given by Berman and Liu [2] and by H. Li, Flandrin and Shu [4].
Theorem B (*Berman and Liu [2]*). *Let D be a strongly connected digraph of order n. Then every Meyniel set M lies in a directed cycle.*

Theorem C (*H. Li, Flandrin and Shu [2]*). *Let D be a digraph of order n and M be a Meyniel set in D. If D is M-strongly connected, then D*

contains a cycle through all the vertices of M.

Theorem C is generalize Theorem B. Theorems B and C also is a common generalization of theorems of Ghouila-Houri and Woodall.

Let D be a digraph of order n. Following [5], we say that a nonempty subset Y of vertices of D satisfies condition A_0 if for every triple of the vertices x, y, z in Y such that x and y are nonadjacent: If there is no arc from x to z, then $d(x) + d(y) + d^+(x) + d^-(z) \geq 3n - 2$. If there is no arc from z to x, then $d(x) + d(y) + d^-(x) + d^+(z) \geq 3n - 2$.
Manoussakis [5] proved the following sufficient condition for hamiltonicity of digraphs.

Theorem D (*Manoussakis [5]*). *Let D be a strongly connected digraph D of order $n \geq 4$. If the vertex set of D satisfies the condition A_0, then D is Hamiltonian.*

H. Li, Flandrin and Shu [4] was put a question to know if this theorem of Manoussakis has a cyclable version. We prove the following theorem which gives a answer for above-mentioned question of H. Li, Flandrin and Shu and is best possible in some sens.

Theorem . *Let D be a digraph of order n and let Y be a nonempty subset of vertices of D. Suppose that D is Y-strongly connected and the subset Y satisfies condition A_0. Then D contains a cycle through all the vertices of Y may be except one.*

References

[1] J. Bang-Jensen, G. Gutin, Digraphs: Theory, Algorithms and Applications, Springer, 2000.

[2] K.A. Berman, X. Liu, Cycles through large degree vertices in digraphs: A generalization of Meyniel's theorem, *J. Combinatorial Theory* **B 74**: 20-27,1998.

[3] S.Kh. Darbinyan, I.A. Karapetyan, On cycles through vertices of large semidegrees in digraphs, *Mathematical Problems of Computer Science*, **39**: 106-118, 2013.

[4] H. Li, E. Flandrin, J. Shu, A sufficient condition for cyclability in directed graphs, *Discrete Mathematics* **307**: 1291-1297, 2007.

[5] Y. Manoussakis, Directed Hamiltonian graphs, *J. Graph Theory* **16**(1):51-59, 1992.

[6] M. Meyniel, Une condition suffisante d'existence d'un circuit Hamiltonien dans un graphe oriente, *J. Combinatorial Theory* **B 14**: 137-147, 1973.

Direct Products and Reduced Products of F-Algebras

S.S. Davidov, J. Hatami

Yerevan State University, Armenia
E-mail: *davidov@ysu.am, jhatami51@gmail.com*

Introduction. Fuzzy approaches to various universal algebraic concepts started with Rosenfeld's fuzzy groups [1]. Since then, many fuzzy algebraic structures have been studied (vector spaces, rings, etc.). Another fuzzy approach to universal algebras was initiated by Bělohl avek and Vychodil [2], who studied the so-called algebras with fuzzy equalities and developed a fuzzy equational logic. The problem of development of algebras with fuzzy operations is formulated in ([2], P: 136).

F-Algebras. In our talk we will use complete residuated lattices $\mathbf{L} = \langle L, \wedge, \vee, 0, 1\rangle$ as the structures of truth values. An $\mathbf{L}$-fuzzy set of X is a mapping $\mu : X \to L$ and the set of all $\mathbf{L}$-fuzzy sets of X is denoted by L^X. An n-ary L-relation of X is a mapping $r : X^n \to L$.

Definition 1. *An $L-$equivalence (fuzzy equivalence) relation E on a set X is a mapping $E : X \times X \to L$ satisfying*

1. $E(x,x) = 1 \quad (Reflexivity)$,
2. $E(x,y) = E(y,x) \quad (Symmetry)$,
3. $E(x,y) \otimes E(y,z) \leq E(x,z) \quad (Transitivity)$,

for every $x, y, z \in X$. An $L-$equivalence E on X where $E(x,y) = 1$ implies $x = y$ will be called an $L-$equality $(fuzzy\ equality)$. $L-$equalities will usually be denoted by $\approx$.

Definition 2. *Let $\approx^{\mathbf{M}}$ be a fuzzy equality on M. An (n+1)-ary fuzzy relation r on a set M is called an n-ary fuzzy operation w.r.t. $\approx^{\mathbf{M}}$ and $\approx^{\mathbf{M^n}}$ if we have the following conditions*
Extensionality:
$(p \approx^{\mathbf{M^n}} p') \otimes (y \approx^{\mathbf{M}} y') \otimes r(p,y) \leq r(p',y') \quad \forall p,p' \in M^n,\ \forall y,y' \in M,$
Functionality:

$$r(p,y) \otimes r(p,y') \leq y \approx^{\mathbf{M}} y' \quad \forall p \in M^n,\ \forall y,y' \in M,$$

Fully- defined:

$$\bigvee_{y\in M} r(p,y)=1 \quad \forall p\in M^n,$$

where $(a_1,\cdots,a_n)\approx^{M^n}(b_1,\cdots,b_n)=\bigwedge_{i=1}^{n}(a_i\approx^{M}b_i)$. *We say that* ρ *is a fuzzy operation on* M *with arity* n.

Definition 3. *An algebra (structure) with fuzzy operations of type* $\tau=\langle\approx,R\rangle$*, (consisting a binary relation symbol* $\approx$ *is called a symbol for fuzzy equality and a set* R *of symbols of operations, and* $\approx\notin R$*), is a triplet* $\mathcal{M}=\langle M,\approx^{\mathcal{M}},R^{\mathcal{M}}\rangle$ *such that*

1. $\approx^{\mathcal{M}}$ *is a fuzzy equality on the set* M*,*

2. $R^{\mathcal{M}}$ *is a set of fuzzy operations on the set* M*.*

Definition 4. *Let* I *be an index set and* F *a filter on* I*. A reduced product of a family* $\{\mathcal{M}_i\mid i\in I\}$ *of algebras with fuzzy operations* $\mathcal{M}_i=\langle M_i,\approx^{\mathcal{M}_i},R^{\mathcal{M}_i}\rangle$ *of type* $\langle\approx,R\rangle$ *is an algebras with fuzzy operations*

$$\prod\nolimits_{i\in I}\mathcal{M}_i=\langle\prod_{i\in I}M_i,\approx^{\prod_{i\in I}\mathcal{M}_i},R^{\prod_{i\in I}\mathcal{M}_i}\rangle \tag{1}$$

such that for every $(n+1)$*-ary fuzzy relation* $r^{\prod_{i\in I}\mathcal{M}_i}\in R^{\prod_{i\in I}\mathcal{M}_i}$ *and* $a_1,\cdots,a_n,y\in\prod_{i\in I}M_i$ *we have*

$$r^{\Pi_{i\in I}\mathcal{M}_i}(a_1,\cdots,a_n,y)=\bigvee_{X\in F}\bigwedge_{i\in X}r^{\mathcal{M}_i}(a_1(i),\cdots,a_n,(i),y(i)),$$

and

$$(a\approx^{\prod_{i\in I}\mathcal{M}_i}b)=\bigvee_{X\in F}\bigwedge_{i\in X}a(i)\approx^{\mathcal{M}_i}b(i)$$

for all $a,b\in\prod_{i\in I}M_i$*. (*$a(i)$ *is the* i*-th coordinate of* a*).*

If $F=\{I\}$*, we obtain the direct product of* F*-algebras.*

In our talk we will present some properties of direct products and reduced products of F-algebras.

References

[1] A. Rosenfeld, Fuzzy groups. *J. Math. Anal. Appl.*, 35(3), 512-517, 1971.

[2] A. Bělohl avek, V. Vychodil. Fuzzy Equational Logic. *Springer, Berlin, Heidelberg.*, 2005.

On the isomorphism of heat operator in spaces with quasi-homogeneous norm

A.A. Davtyan

Yerevan State University, Armenia
E-mail: *davtyan-an@mail.ru*

Let R^n be the Euclidean space with points $x = (x_1, x_2, \ldots, x_n)$, $r = (r_1, r_2, \ldots, r_n)$ a vector with positive components, $\frac{1}{r^*} = \frac{1}{n}\sum_1^n \frac{1}{r_j}$, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)$, where $\lambda_j = \frac{r^*}{r_j}$, $j = 1, 2, \ldots, n$. By $\rho(x)$ we denote the function, positive for $x \neq 0$, defined implicitly by the equality

$$\sum_{i=1}^n x_i^2 \rho^{-2\lambda_i} = 1.$$

It is natural (see [4]) to denote the completion of $C_0^\infty(R^n)$ in the norm

$$\|f\| = \left\| F^{-1}\left(\rho^{r^*}(\xi) F\varphi(\xi)\right)\right\|_p, \quad 1 < p < \infty,$$

by the simbol $\dot{w}_p^r$ (see also [1] and [2]) and called space with quasihomogeneous norm, or space of anisotropic potentials.

If $r^* < \frac{n}{p}$, then $\dot{w}_p^r$ is the spase of functions representable with anisotropic potentials. When $r^* \geqslant \frac{n}{p}$ the spase $\dot{w}_p^r$ is no longer a function space; its elements are classes in which functions that differ by a corresponding polynomial are identified (see [3], [4]).

When $r_1 = r_2 = \cdots = r_n = 0$ we set $\dot{w}_p^r = L_p(R^n)$. The space $\dot{w}_p^{-r}$ is defined as the dual of $\dot{w}_p^r$. Suppose also that $\dot{w}_p^{2s,2s,\ldots,s} = \dot{w}_p^{2s,s}$, and $\dot{w}_{p,+}^r$ through the subspace of $\dot{w}_p^r$ consisting of factor-classes containing a distribution with support in the $\bar{R}_+^n = \{(x_1, x_2, \ldots, x_{n-1}),\ t \geqslant 0\}$.

Heat operator denoted by

$$T = -\Delta_x + \frac{\partial}{\partial t},$$

where Δ_x is the Laplasian applied to the variables $x_1, x_2, \ldots, x_{n-1}$. As shown in [5] operator

$$T^k : \dot{w}_2^{2s,s} \longrightarrow \dot{w}_2^{2(s-k),s-k},$$

is an isomorphism. A special property of T^k operators is that their symbols can be analitically continued to the last variable in the lower half-plane. This allows us to prove the following

Theorem. T^k operator is an isomorphism from $\dot{w}_{2,+}^{2k,k}$ to $L_{2,+}$.

References

[1] Sobolev S.L. Introduction to the theory of quadrature formulas, "Nauka", Moscow, 1974.

[2] Lizorkin P.I. About the Riesz potentials of arbitrary order, Proc. Steklov Inst. Math. 105(1969), p. 174-197.

[3] Pryde A.J. Spaces with homogeneous norms, Bull.Austral.Math.Soc., v. 21, 1980, p. 189-205.

[4] DavtyanA.A. Sobolev-Liouville spaces with quasihomogeneous norm, Izv.Vyssh.Uchebn. Zaved.Mat., 1986:5, p. 82-84.

[5] Davtyan A.A. Anisotropic potentials, their inversion, and some applications, Soviet Math. Dokl. vol. 32(1985), No. 3, p. 717-721.

Orientation-dependent chord length distributions and maximal chords

A.G. Gasparyan

Yerevan State University, Armenia
E-mail: *ara1987-87@mail.ru*

Let $R^n (n \geq 2)$ be the n-dimensional Euclidean space, $\mathbf{D} \subset R^n$ be a bounded convex body with inner points, S^{n-1} be the $(n-1)$-dimensional sphere of radius 1 centered at the origin and V_n be n-dimensional Lebesgue measure. We consider a random line which is parallel to $u \in S^{n-1}$ and intersects $\mathbf{D}$, that is an element from:

$$\Omega_1(u) = \{\text{lines which are parallel to } u \text{ and intersect } \mathbf{D}\}.$$

Let $\Pi r_{u^\perp}\mathbf{D}$ be the orthogonal projection of $\mathbf{D}$ on the hyperplane $u^\perp (u^\perp$ is the hyperplane with normal u and passing through the origin). A random line which is parallel to u and intersects $\mathbf{D}$ has an intersection point (denote by x) with $\Pi r_{u^\perp}\mathbf{D}$. We can identify the points of $\Pi r_{u^\perp}\mathbf{D}$ and the lines which intersect $\mathbf{D}$ and are parallel to u. The last means, that we can identify $\Omega_1(u)$ and $\Pi r_{u^\perp}\mathbf{D}$. Assuming that the intersection point x is uniformly distributed over the convex body $\Pi r_{u^\perp}\mathbf{D}$ we can define the following distribution function: The function

$$F(u,t) = \frac{V_{n-1}\{x \in \Pi r_{u^\perp}\mathbf{D} : V_1(g(u,x) \cap \mathbf{D}) < t)\}}{b_{\mathbf{D}}(u)}$$

is called orientation-dependent chord length distribution function of $\mathbf{D}$ in direction u at point $t \in R^1$, where $g(u,x)$ - is the line which is parallel to u and intersects $\Pi r_{u^\perp}\mathbf{D}$ at point x and $b_{\mathbf{D}}(u) = V_{n-1}(\Pi r_{u^\perp}\mathbf{D})$.

The orientation-dependent chord length distribution function of a triangle and an ellipse depends on maximal chord $t_{max}(u)$ in direction u (see [1] and [2]). A natural question arises, in which cases does it exist a function $G(x,y)$ of two variables such that $F(u,t) = G(t_{max}(u),t)$. In [3] a necessary condition for orientation-dependent chord length distribution function as a function of maximal chord is obtained. A class of parallelograms for which the necessary condition is not satisfied is also constructed (see [3]).

References

[1] Gasparyan, A. G. and Ohanyan, V. K. Recognition of triangles by covariogram. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 48 (3): 110-122, 2013.

[2] Harutyunyan, H. S. and Ohanyan, V. K. Orientation-dependent section distributions for convex bodies. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 49 (4): 139-156, 2014.

[3] Gasparyan, A. G. and Ohanyan, V. K. Orientation-dependent chord length distribution as a function of maximal chord. *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 50 (2): 98–106, 2015.

Theory of random processes without "usual" assumptions

K.V. Gasparyan

Yerevan State University, Armenia
E-mail: *kargasp@gmail.com*

Investigations concerning to the theory of random processes for the case when so-called "usual" conditions on a stochastic basis doesn't satisfies, go back to the works of J.L. Doob, P.A. Meyer, C. Dellacherie and many others. The strong martingales in [9], A-martingales in [8] and optional martingales in [1, 2] have been introduced and the stochastic calculation by such martingales have been constructed then. In difference from the classical theory of random processes with "usual" assumptions on a stochastic basis, where considering martingales are càdlàg - processes, the martingales entered above are làglàd processes, i.e. the processes with the paths admitting both one-sided finite limits at the each point $t > 0$. Subsequently the weak convergence of distributions for optional semimartingales and the Central Limit Theorems have been received in [4]. The Limit Theorems together with earlier obtained Strong Laws of Large Numbers for optional martingales were used in statistical applications in [3, 5].

Interest for the theory of random processes without "usual" assumptions on a stochastic basis and, in particular, to its applications in Stochastic Finance was shown recently again (see e.g. [7, 6, 10]).

References

[1] L. Gal'chuk. Optional Martingales. *Math.USSR - Sbornik*, 40(4):435–468, 1981.

[2] L. Gal'chuk. Stochastic Integrals with Respect to Optional Semimartingales and Random Measures. *Theory Probab. Appl.*, 29(1):93–108, 1985.

[3] K. Gasparian. Rao - Cramer Inequality for General Filtered Statistical Models. *Journal of Contemporary Math. Analysis*, 39(2):21–32, 2004.

[4] K. Gasparian. Functional Limit Theorems for O-Semimartingales. *Proceedings of the 8th Seminar on Probab. and Stoch. Processes, University of Guilan, Rusht, Iran*, 20–28, 2011.

[5] K. Gasparian. Regression O-Martingale Models. *Proceedings of the 11th Iranian Statistical Conference, Iran University of Science and Technology, Tehran, Iran*, 192–201, 2012.

[6] K. Gasparian. About Uniform O-Supermartingale Decomposition in Nonstandard Case. *Workshop on Stochastic and PDE Methods in Financial Mathematics, Yerevan, Armenia*, 14–16, 2012.

[7] C. Kühn, M. Stroh. A note on stochastic integration with respect to optional semimartingales. *Electronic Communications in Probab*, 14:192–201, 2009.

[8] E. Lenglart. Tribus des Meyer et théorie des processus. *Seminar Probab. XIV, Strasbourg, Lect. Notes Math.*, 784:500–546, 1980.

[9] J. Mertens. Théorie des processus stochastiques générauxa applications aux surmartingales. *Z. Wahsch.- theorie und Verw. Gebiete*, 22:45–68, 1972.

[10] C. Czichowsky, W. Schachermayer. Strong supermartingales and limits of non-negative martingales. *arXiv:1312.2024v2 [math.PR]*, 32p., 2014.

A unified approach to the curve fitting problem

L. Gevorgyan

Armenian National Polytechnic University, Armenia
E-mail: *levgev@hotmail.com*

Curve fitting is the process of constructing a curve, that has the best fit to a series of data points. Curve fitting can involve either interpolation, where an exact fit to the data is required, or smoothing, in which a "smooth" function is constructed that approximately fits the data. The oldest data fitting algorithm is the Lagrange interpolation formula. For the given data set $M = \{(x_k.y_k)\}_{k=0}^{n}$, where no two x_k are the same find the polynomial of the least degree that at each x_k assumes the corresponding value y_k. This problem is always solvable and the solution is unique. Start by the product

$$\omega(x) = \prod_{k=0}^{n} (x - x_k)$$

and introduce the Lagrange fundamental polynomials

$$l_k(x) = \frac{\omega(x)}{(x - x_k)\,\omega'(x_k)}$$

Finally, the interpolating polynomial is defined by the formula

$$P(x) = \sum_{k=0}^{n} y_k l_k(x).$$

Conceived to serve as a tool in the investigation of functions, interpolation polynomials suffer two serious flaws. The first is the polynomial wiggle, i.e. increasing the degree of the polynomial makes the oscillations very large. Famous Runge's example shows that for the function

$$f(x) = \frac{1}{1 + x^2},\ x \in [-5; 5]$$

the interpolation polynomials constructed by the equidistant nodes tend to infinity in the uniform norm, when the number of nodes grows infinitely. Moreover, for any choice of nodes there exists a function, such that the error (the norm of the difference between the function and the interpolation polynomial) tends to infinity.

For the same knots another interpolation formula may be obtained, introducing Hermite-Fejer basic polynomials

$$h_k(x) = \left(1 - \frac{\omega''(x_k)}{\omega'(x_k)}(x - x_k)\right) l_k^2(x).$$

According to Fejer's theorem the Hermite-Fejer interpolating polynomials, constructed by the nodes, consisting of zeros of the Chebyshev polynomials of the first kind tend to any continuous on $[-1; 1]$ function f.

For interpolation by splines usually are used cubic polynomials, different for each pair of neighbouring nodes, regularized such that at each node the resulting function and its derivatives up to the order two are continuous. All these conditions lead to a diagonally dominated tridiagonal system of linear equations, which is uniquely solvable.

The smoothing idea is implemented in the Bernstein polynomials. The general case of arbitrary interval $[a; b]$ is reduced to $[0; 1]$ and set of weights (Bernstein basis polynomials)

$$b_k(t) = C_n^k t^k (1-t)^{n-k}, t \in [0; 1]$$

are introduced. For any continuous on $[0; 1]$ function f the sequence of Bernstein polynomials

$$B_n(f, t) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) b_k(t)$$

converges uniformly on $[0; 1]$ to f.

We propose a general approach to these problems, including as particular cases the above mentioned interpolation and smoothing algorithms, as well the Bezier curves.

References

[1] Cheney, Ward; Light, Will. A course in approximation theory. Reprint of the 2000 original. Graduate Studies in Mathematics, 101. American Mathematical Society, Providence, RI, 2009. xvi+359 pp. ISBN: 978-0-8218-4798-5

[2] P. J. Davis, *Interpolation and approximation*, Dover, New York, 1975.

[3] P. Lancaster and K. Šalkauskas, *Curve and surface fitting*, Academic Press, London, 1986.

The classic three-body problem in general case as a system of 6th order

A.S. Gevorkyan

Institute for Informatics and Automation Problems, NAS of Armenia,
Institute of Chemical Physics, NAS of Armenia
E-mail: *g_ashot@sci.am*

Introduction. A large class of elementary atomic-molecular and nuclear processes can be described in the framework of representations the three and four-body (a few-body). Let us note that a few-body problems, both classical and quantum, are generally non-integrable systems and respectively the main method of their studies is the numerical simulation. For applied problems it is often important to know probabilities and cross-sections elementary reactions over a wide range of initial parameters, that implies the large-scale calculations, the development of new more effective algorithms always remains an important challenge. On the assumption of the previously mentioned it becomes apparent, that the reduction of dimensionality of the dynamical problem is a very important problem. If we talk about the classical three-body problem then as it is well-known, the general problem in the phase space is described by the $8th$ ordinary differential equations of first order (the system of $8th$ order). Note, that this system of nonlinear equations is rather complex and its numerical simulation from itself represents non-trivial problem. The calculations especially become complicated when we want to solve the three-body collision problem with consideration of the multichannel scattering (see scheme below);

$$A+(BC)\to\left\{\begin{array}{c} A+(BC),\\ C+(AB),\\ B+(AC),\\ A+B+C,\\ (ABC)^*\to\left\{\begin{array}{c} C+(AB),\\ B+(AC),\\ A+B+C,\\ (ABC)^{**},\end{array}\right.\end{array}\right.$$

where A, B, and C denote colliding particles, $(ABC)^*$ and $(ABC)^{**}$ respectively denote transient (resonant) complexes which are forming in result of three-body collision.

We have proved that the general three-body problem may be formulated as the problem of geodesic flows on the $6D$ manifold; $M \cong \mathcal{M}_t \otimes S^3$, where $\mathcal{M}_t$ denotes tangent bundle (the $3D$ hypersurface of an energy which is defined by diagonal matrix $g_{ij}(\{x\}) = \big(E - U(\{x\})\big)\delta_{ij}$ where $\{x\}$ the set of three coordinates, E and $U(\{x\})$ respectively the total energy and interaction potential of the body-system), $SO(3)$ is the space of the rotation group. This allows to find a new type symmetry and to implement more complete integration of system in result of which the initial problem is reduced to the system of the $6th$ order [1]. It is important to note, that on the way to the proof of reduction of three-body dynamical system to the system of 6th order, the Poincare conjecture about of homeomorphism between a closed $3D$ manifold and the standard sphere S^3 is proved. As it is shown, the dynamical system that arises in result of reducing the initial three-body problem is described with the help of exact Hamiltonian. The last allows developing the symplectic algorithm for a numerical simulation of considered problem. Finally, the multichannel chemical and nuclear reactions often pass via of formation of transition complex (see the scheme), that makes behavior of geodesic flows on curved space similar to phenomena of turbulence in fluids. This obviously gives us a good opportunity to use already well-established, high-performance program-package from the field of modeling turbulence.

References

[1] A. S. Gevorkyan, Journal of Physics: Conference Series **496** (2014) 012030. doi:10.1088/1742-6596/496/1/012030

Calculations from the first principles, and reduction of NP hard problem to the P problem on the example of $1D$ spin-glass

A.S. Gevorkyan, V.V. Sahakyan

Institute for Informatics and Automation Problems, NAS of Armenia,
Institute of Chemical Physics, NAS of Armenia
E-mail: *g_ashot@sci.am*

Introduction. Not so long ago, the famous physicist Stephen Hawking in his "millennium" interview (San Jose Mercury News, January 23, 2000) said, "I think the next century will be the century of complexity. As we now see his prediction turned out to be prophetic. Up to now there is no clear definition of what complexity means. However, a characterization of what can be regarded as a complex is possible. In a number of scientific fields, "complexity" has a precise meaning. Furthermore, in a number of scientific fields, "complexity" has a precise meaning. In particular it concerns to the field of computational complexity theory where the amounts of resources required for the execution of algorithms is studied rigorously. The most popular types of computational complexity are classified by complexity classes P and NP. Recall that the P class includes problems that can be solved in polynomial time on the Turing machine, whereas the class of NP problems on the same machine is impossible to solve in a polynomial time. In regard of this arises an important open problem, namely, the problem of equivalence of classes NP and P.

In the present work we continue study on problem of computational complexity of spin-glasses [1]. In particular we have studied the classical $1D$ Heisenberg spin glasses assuming that spins are spatial. The system of recurrence equations are derived by minimization of the nearest-neighboring Hamiltonian in nodes of $1D$ lattice. It is proved, that there is probability that in each node of lattice the solution of recurrence equations can bifurcate. In result of this, performing a consecutive node-by-node calculations, on the *n-th* step instead of a single stable spin-chain we receive a set of spin-chains which form Fibonacci subtree (graph). Theoretically the complexity of computation of one graph is assessed, it is equal $\propto 2^n K_s$, where n and K_s denote the subtree's height (the length of spin-chain) and Kolmogorov's complexity of a string (the branch of subtree) respectively. It is shown that the statistical ensemble may be represented as a set of a random graphs, where the computational complexity of each of them is $\mathbb{NP}$ hard. It is proved that all strings of the ensemble have same weights. That allows, in the limit of statistical equilibrium with predetermined accuracy to reduce $\mathbb{NP}$ hard problem to the $\mathbb{P}$ problem with complexity $\propto NK_s$, where N is the number of spin-chains in the ensemble. As it is shown by comparing of statistical distributions of different parameters which are performed by using of $\mathbb{NP}$ and $\mathbb{P}$ algorithms, the coincidence of the corresponding curves is ideally (see Fig. 1). The latter allows to claim that all parameters and characteristic distributions of statistical ensemble can be calculated from the first principles of classical mechanics without using any additional considerations.

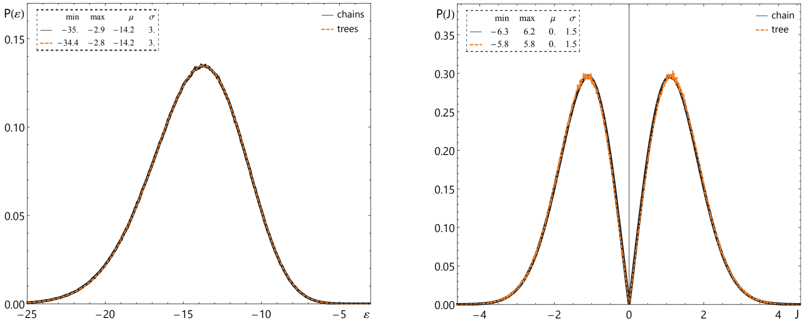

Figure 1: *The distributions of the energy and coupling constants (spin-spin interaction constants) into the statistically equilibrium ensemble consisting of spin-chains of the length 20.The black curves denote the distributions which have calculated by P algorithm, while whites are calculated by NP algorithm.*

Finally, for the partition function we propose a new representation in the form of one dimensional integral by the energy distribution of spin-chains. Let us note that this representation for the partition function does not include contributions of spin-chains configurations which does not satisfy to the first principles of classical mechanics.

References

[1] A. S. Gevorkyan et al., *On reduction of NP hard problem to the P problem on the example of 1D spin glasses*, 13 - 21 August, 2014, Seoul, Korea.

Linear differential equations with constant powers

H.G. Ghazaryan, V.N. Margaryan

Russian-Armenian Slavonic University, Armenia
Institute of Mathematics of National Academy of Sciences, Armenia
E-mail: *haikghazaryan@mail.ru*

We say that the differential operator $R_1(D)$ (the polynomial $R_1(\xi)$) with constant coefficients is **more powerful** than the differential operator $R_2(D)$ (the polynomial $R_2(\xi)$) and write $R_2 < R_1$ if for some constant $C > 0$ $|R_2(\xi)| \le C\,[|R_1(\xi)|+1]$ for all $\xi \in R^n$. If $|R_2(\xi)|/[|R_1(\xi)|+1] \to 0$ as $|\xi| \to \infty$ we write $R_2 << R_1$.

Let $P(x,D) = \sum\limits_{\alpha} \gamma_\alpha(x)\, D^\alpha$ be a linear differential operator with coefficients, difined on E^n and $P(x,\xi) = \sum\limits_{\alpha} \gamma_\alpha(x)\, \xi^\alpha$ be its characteristic polynomial (complite symbol), where for each $x \in E^n$ the sum extends over a finite collection of multi - indices $(P,x) = \{\alpha \in N_0^n; \gamma_\alpha(x) \ne 0\}$.

An operator $P(x,D)$ (a polynomial $P(x,\xi)$) we call **formally almost hypoelliptic** in E^n, if 1) $P(x,D)$ have **constant power** in E^n (see [1] or [2]), i.e. if $P(x^1,\xi) < P(x^2,\xi)$ for arbitrary fixed $x^1, x^2 \in E^n$ and 2) operator $P(x^0,D)$ with constant coefficients is **almost hypoelliptic** for any $x^0 \in E^n$, i.e. $D^\nu P(x^0,D) < P(x^0,D)$ for all $\nu \in N_0^n$.

Let I_n denote the set of polynomials $R(\xi) = R(\xi_1,...,\xi_n)$ with constant coefficients such that $|R(\xi)| \to \infty$ as $|\xi| \to \infty$. In [3] it has founded some conditions under which an almost hypoelliptic polynomial $R \in I_n$.

Denote by $A = A(E^n)$ a class of operators $P(x,D)$ with coefficients in C^∞ and with constant power in E^n, satisfying conditions: there exists a point $x^0 \in E^n$ such that operator $P(x,D)$ represents in form

$$P(x,D) = P_0(D) + \sum_{j=1}^{r} a_j(x) P_j(D), \tag{1}$$

where $P_0(D) = P(x^0,D)$ and 1) $a_j \in C^\infty$ $(j = 1,...,r)$, 2) $P_0 \in I_n$ and $P_j << P_0$ $(j = 1,...,r)$, 3) for any $\nu \in N_0^n$ there exists a number $c_\nu > 0$ such that $|D^\nu a_j(x)| \le c_\nu$ $(j = 1,...,r)$ for all $x \in E^n$, 4) there exists a number $c > 0$ such that $|\xi| \le c\,[|P_0(\xi)| + 1]$ for all $\xi \in R^n$.

It is easily to seen that a) any operator $P \in A$ is formally almost hypoelliptic, b) for each point $x^0 \in E^n$ any operator P with constant power represents in form $P(x,D) = P_0(D) + \sum_{j=0}^{r} a_j(x)P_j(D)$ (note that in representacion (1) $a_0(x) \equiv 0$), where $P_0(D) = P(x^0, D)$ and the coefficients $\{a_j(x) = a_j(x, x^0)\}$ are uniquely determined, vanish at x^0, have the same differentiability properties as the coefficients of $P(x,D)$ and $P_0, P_1, ..., P_r$ is a basis in the finite dimensional vector space of operators with constant coefficients which are less powerful than $P_0(D)$ (for operators with constant strength by L. Hormander see [4]), c) any non - degenerate by V.P.Mikhailov operator $P(x,D)$ with complite Newton polyhedron $\Re(P) \equiv \Re(x,P)$ (see [5]) and some degenerate operators (see, for instance, [2]) satisfies the condition 4) of the class A.

Let $g(x)$ be a weight function such that $\sigma^{-1}e^{-|x|} \le g(x) \le \sigma e^{-|x|}$ for a number $\sigma > 0$ and $g_\delta(x) = g(\delta\, x)$, $||u||_{L_{2,\,\delta}} = \{u; ||u\, g_\delta||_{L_2} < \infty\}$, $H(P_0,\,\delta) = \{u; ||u||_{H(P_0,\delta)} = ||u||_{L_{2,\,\delta}} + ||P_0(D)u||_{L_{2,\,\delta}} < \infty\}$, $D(P) = \{u \in S^{'}; P(x,D)u = 0\}$. We prove the following result

Theorem 1. *Let $P \in A$, then there exists a number $\Delta > 0$ such that $D(P) \cap H(P_0,\,\delta) \subset C^\infty$ for any $\delta \in (0,\Delta)$.*

References

[1] F. Treves, Relations de domination entre operateurs differentiels. Acta Math., v.101, 1 - 139, 1959.

[2] G.G.Kazaryan, Permanent strength operators with lower estimates trough derivatives and formally hypoelliptic operators. Analysis Mathematica, v.3, no.4, 1977.

[3] H.G.Ghazaryan, V.N.Margaryan, On increase at infinity of the almost hypoellipticpolynomials. Eurasian Math. Journal, v.4, no. 4, 30 -42, 2013.

[4] L. Hörmander, " The Analysis of Linear Partial Differential Operators". Springer ,1983

[5] V.P.Mikhailov, On the behaviour at infinity of a class of polynomials. Proc. Steklov Inst. Math. v.91, 59 - 81, 1967.

The balanced hyperidentities in invertible algebras and semigroups

H. Ghumashyan

Vanadzor State University, Armenia
E-mail: *hgumashyan@mail.ru*

The present paper is devoted to the study of balanced $\{2,3\}$-hyperidentities of the length of four in invertible algebras and $\{3\}$-hyperidentities of associativity in semigroups.

The following second order formula is called hyperidentity:

$$\forall X_1,\ldots,X_m \forall x_1,\ldots,x_n (W_1 = W_2), \tag{1}$$

where $X_1,\ldots,X_m$ are the functional variables, and $x_1,\ldots,x_n$ are the object variables in the words (terms) W_1, W_2. Usually, a hyperidentity is specified without universal quantifiers of the prefix of the equality: $W_1 = W_2$. According to the definition, the hyperidentity $W_1 = W_2$ is said to be satisfied in the algebra (Q,Σ) if this equality holds when every functional variable X_i is replaced by any arbitrary operation of the corresponding arity from Σ and every object variable x_j is replaced by any arbitrary element from Q.

If the arities of the functional variables are: $|X_1| = n_1,\ldots,|X_m| = n_m$, then the hyperidentity $W_1 = W_2$ is called $\{n_1,\ldots,n_m\}$-hyperidentity.

A hyperidentity is balanced if each object variable of the hyperidentity occurs in both parts of the equality $W_1 = W_2$ only once. A balanced hyperidentity is called first sort hyperidentity, if the object variables on the left and right parts of the equality are ordered identically. The number of the object variables in a balanced hyperidentity is called length of this hyperidentity.

The algebra (Q,Σ) with the binary and ternary operations is called $\{2,3\}$-algebra. A $\{2,3\}$-algebra is called non-trivial, if the sets of its binary and ternary operations are not singleton.

The present paper aims at classifying of the balanced $\{2,3\}$-hyperidentities of length four in invertible algebras and the description of the invertible algebras in which these hyperidentities hold, as well as at the description of the semigroups that polynomially satisfy ternary associative hyperidentities.

The following main results will be proved in the talk.

1. The balanced first sort $\{2,3\}$-hyperidentities of length four in non-trivial invertible algebras are classified;

2. The invertible $\{2,3\}$-algebras with a binary group operation and with the balanced first sort $\{2,3\}$-hyperidentities of the length four are described;
3. The invertible $\{2,3\}$-algebras with ternary group operation and with the balanced first sort $\{2,3\}$-hyperidentities of length four are described;
4. The classes of the semigroups, which polynomially satisfy the associative $\{3\}$-hyperidentities are described.

References

[1] H.O.Pflugfelder, *Quasigroups and Loops: Introduction*, Helderman Verlag Berlin, 1990.

[2] Yu.M. Movsisyan, *Introduction to the theory of algebras with hyperidentities*, Yerevan State University Press, Yerevan, 1986. (Russian)

[3] Yu.M.Movsisyan, *Hyperidentities and hypervarieties in algebras*, Yerevan State University Press, Yerevan, 1990 (Russian).

[4] Yu.M. Movsisyan, Hyperidentities in algebras and varieties, *Uspekhi Matematicheskikh Nauk*, 53 (1998), pp. 61–114. English translation in Russian Mathematical Surveys 53(1998), pp. 57–108.

[5] Yu.M. Movsisyan, Hyperidentities and hypervarieties, *Scientiae Mathematicae Japonicae*, 54(3), (2001), 595–640.

[6] M.Hazewinkel (Editor), *Handbook of algebra*, Vol. 2, North-Holland, 2000.

[7] G.M.Bergman, *An invitation on general algebra and universal constructions*, Second edition, Springer, 2015.

[8] J.D.H. Smith, On groups of hypersubstitutions, *Algebra Universalis*, 64, (2010), 39–48.

[9] K. Denecke, J. Koppitz, *M-solid varieties of Algebras*. Advances in Mathematic, 10, Spriger-Science+Business Media, New York, 2006

[10] K. Denecke, S.L. Wismath, *Hyperidentities and Clones*. Gordon and Breach Science Publishers, 2000.

On a solutions of one class of almost hippoelliptic equations

G.H. Hakobyan

Yerevan State University, Armenia
E-mail: *gaghakob@ysu.am*

Introduction. We prove that if $P(D) = P(D_1, D_2) = \sum_\alpha \gamma_\alpha D_1^{\alpha_1} D_2^{\alpha_2}$ is almost hippoelliptic regular operator, then for enough small $\delta > 0$, all solutions of equation $P(D)u = 0$ from $L_{2,\delta}(R^2)$ are entire analytical functions.

Section 1. We use standard notations: N is the set of natural numbers; $N_0 = N \cup \{0\}$; $N_0^n = N_0 \times \cdots \times N_0$ is the set of n-dimensional multi-indices; E^n and R^n are n-dimensional real coordinate spaces $x = (x_1, \ldots, x_n)$ and $\xi = (\xi_1, \ldots, \xi_n)$, respectively.

Let $B = \{\alpha^k\}$ be finite set of points from N_0^n. Minimal convex polyhedron $\Re = \Re(B) \subset R_+^n$ including $B \cup \{0\}$ we call as characteristic polyhedron or the Newton polyhedron of set B. We call polyhedron $\Re$ as regular if $\Re$ has vertex at the origin, vertices on each axes apart from the origin, and all outer (relative to $\Re$) normals of $(n-1)$-dimensional faces of $\Re$ have nonnegative coordinates. We call polyhedron $\Re$ as completely regular if all outer normals of such faces have only positive coordinates.

Let $P(D) = \sum_\alpha \gamma_\alpha D^\alpha$ be linear differential operator with constant coefficients and $P(\xi) = \sum_\alpha \gamma_\alpha \xi^\alpha$ be the corresponding symbol (characteristic polynomial), where summation is performed over the following finite set of multi-indices $(P) = \{\alpha \in N_0^n, \gamma_\alpha \neq 0\}$.

Characteristic polyhedron $\Re = \Re(P)$ of set (P) we call as characteristic polyhedron of operator $P(D)$ (polynomial $P(\xi)$).

Definition 1. *[1, 2] Operator $P(D)$ with polyhedron $\Re = \Re(P)$ is called regular if there exists constant $C > 0$ such that*

$$\sum_{\alpha \in \Re \cap N_0^n} |\xi^\alpha| \leq C(|P(\xi)| + 1), \ \forall \xi \in R^n.$$

Definition 2. *[3] Operator $P(D)$ (polynomial $P(\xi)$) is called almost hippoelliptic if there exists constant $C > 0$ such that for any $\beta \in N_0^n$*

$$|D^\beta(\xi)| \leq C(|P(\xi)| + 1), \ \forall \in R^n.$$

For $\delta > 0$ we put

$$N(P, \delta) = \{u; u \in L_{2,\delta}(E^2), P(D)u = 0\},$$

where

$$L_{2,\delta}(E^2) = \{f; fe^{-\delta|x|} \in L_2(E^2)\}.$$

Let

$$P_0(D) = P_0(D_1, D_2) = \sum_{\alpha \in N_0^2} \gamma_\alpha D_1^{\alpha_1} D_2^{\alpha_2}$$

is a regular operator with characteristic polyhedron

$$\Re(P_0) = \{\nu \in R_+^2, \nu_1 \leq m_1, \nu_2 \leq m_2\},$$

where $m_1, m_2 \in N_0$. Obviously, $\Re(P_0)$ is a regular polyhedron.

We denote by $A_0(E^2)$ the set of entire analytical functions of real variables (x_1, x_2).

Theorem 1. *For any compact set $K \subset E^2$ and for any function $u \in N(P_0, \delta)$ the following estimate is holds*

$$\sup_{x \in K} |D^\alpha u(x)| \leq C^{|\alpha|+1}, \ \forall \alpha \in N_0^2,$$

where $C = C(K, u)$ is some constant and $\delta > 0$ is sufficiently small.

From here, we conclude that for sufficiently small $\delta > 0$ the following holds

$$N(P_0, \delta) \subset A_0.$$

We prove also that for sufficiently small $\delta > 0$ and for any $f \in \Gamma_\delta^a(E^2)$, $a > 1$ the following holds

$$N(P_0, f, \delta) \equiv \{u, u \cdot e^{-\delta|x|} \in L_2(E^2), P(D)u = f\} \subset \Gamma_\delta^a(E^2),$$

where

$$\Gamma_\delta^a(E^2) = \{f; \|D^\alpha f \cdot e^{-\delta|\alpha|}\|_{L_2} \leq C^{|\alpha|+1} |\alpha|^{a|\alpha|}\}.$$

References

[1] V.P. Mikhailov, On behavior at infinity of a class of polinomials, *Trydi MIAN SSSR*, 150:143-159, 1965.

[2] S. Gindikin, L. Volevich, The method of Newtons Polyhedron in the theory of PDE, *Kluwer*, 1992.

[3] G. G. Kazaryan, *On almost hippoelliptic polynomials*, Reports of the Russian Academy of Sciensec, 398(6), pp. 701–703, 2004.

Neyman-Pearson Lemma for Multiple Fuzzy Hypotheses Testing with Vague Data

E.A. Haroutunian

Institute for Informatics and Automation Problems of NAS, Armenia
E-mail: *evhar@ipia.sci.am*

There are a few studies about multiple hypotheses, the overwhelming majority of publications is dedicated to the case of two hypotheses [5]. Multiple hypotheses testing is an important area in statistical inference with wide applications in many scientific and practical fields [2-4]. Many of decisions in the real world are made in a fuzzy environment. Fuzzy decision problems are studied following the theory founded by Zadeh[7-9].

There are many investications concerning decision problems, hypotheses testing and Neyman-Pearson lemma involving uncertainty with application of fuzzy set theory [6].

We presented generalization of the Neyman-Pearson lemma for more than two hypotheses in conventional formulation in [1] and now state the Lemma in terms of fuzzy statistics. For neccessary definitions of fuzzy theory notions we refer to [6].

Theorem. Let $\widetilde{X} = (\widetilde{X}_1, ..., \widetilde{X}_N)$ be a fuzzy-valued random sample with observed values $\widetilde{X} = (\widetilde{x}_1, ..., \widetilde{x}_N)$ and density $f(\widetilde{x}, \theta)$, where $\theta \in \Theta$ is a parameter. For testing fuzzily formulated three hypotheses

$$H_1 : \theta \quad \text{is} \quad H_1(\theta),$$

$$H_2 : \theta \quad \text{is} \quad H_2(\theta),$$

$$H_3 : \theta \quad \text{is} \quad H_3(\theta),$$

for preassigned positive numbers T_1, T_2 any test with fuzzy test function

$$\Phi(\widetilde{\mathbf{x}}) =$$

$$\begin{cases} 1, \text{ if } \min\left(\widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_2(\widetilde{\mathbf{x}}), \widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_3(\widetilde{\mathbf{x}})\right) \geq T_1, \\ 2, \text{ if } \min\left(\widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_2(\widetilde{\mathbf{x}}), \widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_3(\widetilde{\mathbf{x}})\right) < T_1 \,, \min\left(\widetilde{H}_2(\widetilde{\mathbf{x}})/\widetilde{H}_3(\widetilde{\mathbf{x}})\right) \geq T_2, \\ 3, \text{ if } \min\left(\widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_2(\widetilde{\mathbf{x}}), \widetilde{H}_1(\widetilde{\mathbf{x}})/\widetilde{H}_3(\widetilde{\mathbf{x}})\right) < T_1 \,, \min\left(\widetilde{H}_2(\widetilde{\mathbf{x}})/\widetilde{H}_3(\widetilde{\mathbf{x}})\right) < T_2, \end{cases}$$

is optimal in the sense that it has error probabilities $\alpha_{l|m}$ $l, m = \overline{1,3}$, and for each other test with the corresponding error probabilities $\beta_{l|m}$, $l, m = \overline{1,3}$,

$$\text{if} \quad \beta_{1|1} \leq \alpha_{1|1}, \quad \text{then} \quad \beta_{1|2} \geq \alpha_{1|2}, \quad \text{or} \quad \beta_{1|3} \geq \alpha_{1|3},$$

and

$$\text{if} \quad \beta_{2|2} \leq \alpha_{2|2}, \quad \text{then} \quad \beta_{2|3} \geq \alpha_{2|3}.$$

Acknowledgement. This work was supported in part by SCS of MES of RA under Thematic Program No SCS 13–1A295.

References

[1] E. A. Haroutunian, "Neyman-Person prnciple for more than two hypotheses", *Abstracts of Armenian Mathematical Union Annual Session Dedicated to 90Anniversary of Rafael Alexandrian, Yerevan,* pp. 49-50, 2013.

[2] E. A. Haroutunian, M. E. Haroutunian, A. N. Haroutunian, "Reliability criteria in information theory and in Statistical Hypothesis Testing", *Foundation and Trends in Communication and Information Theory,* vol. 4, nos. 2-3, pp. 97-263, 2007.

[3] E. A. Haroutunian, "Reliability in multiple hypothese, testing and identification problems", in *Data Fusion for Situation Monitoring Trident Detection, IOS Press,* pp. 189-201, 2005.

[4] E. A. Haroutunian, P. M. Hakobyan,"Multiple objects: Error exponents in hypotheses testing and identification", *Lecture Notes in Computer Science*, vol. 7777, Springer Verlag, pp. 313-345, 2013.

[5] J. Neyman, E. S. Pearson, "On the problem of the most efficient tests of statistical hypotheses", *Phil. Trans. Roy. Soc. London, Ser. A*, vol. 231, pp. 289-337, 1933.

[6] H. Torabi, J. Behboodian, S. M. Taheri, "Neyman-Pearson lemma for fuzzy hypotheses testing with vague data", *Metrika,* vol. 64, pp. 239-304, 2006.

[7] L. A. Zadeh, "Fuzzy sets", *Information and Control*, vol. 8, pp. 338-353, 1965.

[8] L. A. Zadeh, "Probability measures of fuzzy events", *Journal of Mathematical Analysis and Application*, vol. 23, no. 2, pp. 421-427, 1968.

[9] L. A. Zadeh, "Computation with imprecise probabilities", *Proceeding of IPMV'08 Tovemolinos (Malaga)*, June 22-27, 2008.

On Neyman-Pearson Testing for a Pair of Independent Objects

E.A. Haroutunian, P.M. Hakobyan

Institute for Informatics and Automation Problems of NAS, Armenia
E-mail: *eghishe@sci.am, par_h@ipia.sci.am*

Introduction. The Neyman-Pearson lemma [1] plays a central role in the theory and practice of statistics. For the case of multiple hypotheses it is considered in [2].

Here we discuss Neyman-Pearson hypotheses testing principle for a model consisting of two independent objects. This model was proposed by Ahlswede and Haroutunian [1]. The characteristics of the objects are independent random variables (RVs) X_1 and X_2 taking values in the same finite set $\mathcal{X}$. So, considered model is described by the random vector (X_1, X_2), which assumes values $(x^1, x^2) \in \mathcal{X} \times \mathcal{X}$. It is supposed that two probability distributions $G_m = \{G_m(x),\ x \in \mathcal{X},\ m = 1, 2$ are known and each object independently follows to one of them. So, there are four hypothetical probability distributions $G_i \circ G_j(x^1, x^2) = \{G_i(x^1)\dot{G}_j(x^2),\ \ (x^1, x^2) \in \mathcal{X}\times\mathcal{X}\}$, $i, j = 1, 2$, for random vector (X_1, X_2).

Let $(\mathbf{x_1}, \mathbf{x_2}) = ((x_1^1, x_1^2), ..., (x_n^1, x_n^2), ..., (x_N^1, x_N^2))$, $x_n^i \in \mathcal{X}$, $i = \overline{1, 2}$, $n = \overline{1, N}$, be a sequence of results of N independent observations of the vector (X_1, X_2). It is necessary to detect unknown PDs of the pair of objects on the base of observed data. The test can be defined by division of the sample space $\mathcal{X}^N \times \mathcal{X}^N$ on 4 disjoint subsets $\mathcal{B}_{i,j}^N$, $i, j = \overline{1, 2}$. The set $\mathcal{B}_{i,j}^N$ consists of all vectors $(\mathbf{x_1}, \mathbf{x_2})$ for which the hypothesis $G_i \circ G_j$ is adopted.

Let $\alpha_{l_1,l_2|m_1,m_2} = G_{m_1}^N \circ G_{m_2}^N(\mathcal{B}_{l_1,l_2}^N)$, $(l_1, l_2) \neq (m_1, m_2)$, $l_i, m_i = 1, 2$, $i = 1, 2$ be the probability of the erroneous acceptance $G_{l_1} \circ G_{l_2}$ by the test provided that $G_{m_1} \circ G_{m_2}$ is true. When a true distribution $G_{m_1} \circ G_{m_2}$, $m_1, m_2 = 1, 2$ is rejected the error probability is $\alpha_{m_1,m_2|m_1,m_2} = \sum\limits_{(l_1,l_2)\neq(m_1,m_2)} \alpha_{l_1,l_2|m_1,m_2}$.

Neyman-Pearson Testing for a Pair of Independent Objects.

We consider Neyman-Pearson testing for this model with two approaches: a) *direct method* and b) *renumbering method.* Our aim is to compare the corresponding error probabilities of these two approaches and find the best method.

a) *Direct approach.* Error probabilities of separate tests $\alpha_{l_1|m_1}^*$, $l_1, m_1 = 1, 2$ and $\alpha_{l_2|m_2}^*$, $l_2, m_2 = 1, 2$ of the first and the second objects respectively can be obtained according to Neyman-Pearson lemma for given numbers T^I and T^{II}. For the model joint error probability will be $\alpha^*_{l_1,l_2|m_1,m_2} = \alpha^*_{l_1|m_1}\alpha^*_{l_2|m_2}$, for $l_i \neq m_i$, $l_i, m_i = 1,2$, $i = 1,2$ and $\alpha^*_{l_1,l_2|m_1,m_2} = \alpha^*_{l_i|m_i}(1-\alpha^*_{l_j|m_j})$ for $l_i \neq m_i$, $l_j \neq m_j$, $l_i, m_i = 1,2$, $i \neq j$, $i = 1,2$.

b)*Renumbering approach.* We have to renumber pairs of hypotheses and we have to apply Neyman-Pearson lemma for 4 hypotheses, which is investigated in [2]. Hence, for given positive values $\alpha^*_{1,1|1,1}$, $\alpha^*_{1,2|1,2}$ and $\alpha^*_{2,1|2,1}$ we can chose numbers T_1, T_2, T_3 and sets $\mathcal{A}_{i,j}$, $i,j = \overline{1,2}$.

Let us assume that we have found the error probabilities by direct approach. By considering the renumbering approach we will find all other error probabilities. In [2] it is proved that these error probabilities are the smallest, hence we can insist that renumbering approach is not worse that the direct one.

Conclusion. We conclude with some observations and directions for future work. The analogical result can be obtained for the models for which RVs X_1 and X_2 have values from the different sets and there are different lists of hypothetical probability distributions for the first and the second objects. It is desirable to make calculations for practical examples showing what is the complexity of realization for each of two considered methods.

Acknowledgement. This work was supported in part by SCS of MES of RA under Thematic Program No SCS 13–1A295.

References

[1] R. Ahlswede, and E. Haroutunian. On logarithmically asymptotically optimal testig of hypotheses and identification. *Lecture Notes in Computer Science, "General Theory of Information Transfer and Combinatorics"*, 4123:462–478, 2006.

[2] E. Haroutunian, P. Hakobyan. Neyman-Pearson principle for more than two hypotheses. *Mathematical Problems of Computer Science*, 40:34–38, 2013.

Rank test procedures using weak and threshold copula function

E.A. Haroutunian, I.A. Safaryan

Institute for Informatics and Automation Problems of NAS, Armenia
E-mail: *evhar@ipia.sci.am, irinasafaryan@yandex.ru*

1.Statement of problem.
Let $\{(X_n, Y_n)\}_{n=1}^N$ be a chronologically ordered two-dimensional random sequence, statistical properties of which change in some unknown moment (change-point). We consider $\{(X_n, Y_n)\}_{n=1}^N$, as a random sample of random vector (X, Y) with common distribution function $F(x, y)$ and continuous marginals $F_X(x)$ and $F_Y(y)$. Then dependence between random variables (RV's) X and Y can be expressed in unique manner by copula function $C(u, v)$. Recall that copula of X and Y is defined by relation

$$C(F_X(x), F_Y(y)) = F(x, y).$$

We denote by $C^{(n)}(u, v)$ copula of (X_n, Y_n) and by $n_\lambda = [\lambda N]$ for $\Delta < \lambda \leq 1 - \Delta$, $0 < \Delta < \frac{1}{2}$ - change point. Our aim is to test hypotheses for each $n = \overline{1, N}$

$$H_0 : C^{(n)}(u, v) = C(u, v)$$

under

$$H_1 : C^{(n)}(u, v) = I\{n \leq n_\lambda\}C_1(u, v) + I\{n > n_\lambda\}C_2(u, v)$$

where $I\{A\}$ is indicator of the event A and

$$C_1(u, v) \neq C_2(u, v).$$

2. Change moment detection under some prior assumptions.

a) Prior information about possible distinctions between $C_1(u, v)$ and $C_2(u, v)$ are absent. Nonparametric test for this case sugested in [1] is based on the multivariate modification of Kolmogorov-Smirnov test statistic.
b) The copulas $C_1(u, v)$ and $C_2(u, v)$ belong to the same family of one-parametric copulas and differ only in the parameter values, such as the Farlie-Gumbel-Morgenstern families presented by Nelsen [2].
c) The copulas $C_1(u, v)$ and $C_2(u, v)$ belong to different one-parametric families and differ both in the functional type and in parameter values.

For cases b) and c) we suggested an algorithm that allows reducing the detection of changes in copula function occuring in unknown change-point n_λ to testing homogeneity of one RV, for instance Y, with respect to another RV X [3].

The algorithm is based on rank tests statistics and is applied to analysis of real data in [4] and [5].

In the report model examples are given for the case b) when the suggested method cannot be applied under some value of parameter λ. In the case c) the method can be applied provided the copula $C_1(u, v)$ corresponds to a relatively weak dependence, while the copula $C_2(u, v)$ expresses a stronger dependence (the Frank copula, for example).

Acknowledgement. This work was supported in part by SCS of MES of RA under Thematic Program No SCS 13–1A295.

References

[1] B. E. Brodsky, G. I. Penikas and I. A. Safaryan, "Detecting structural changes in the copula models", (in Russian), *Applied Econometrics, 4(16)*, pp. 3-16, 2009.

[2] R. V. Nelsen, *An Introduction to Copulas*, Springer, New York, 2006.

[3] E. A. Haroutunian and I. A. Safaryan, "Copulas of two-dimensional threshold models", *Mathematical Problems of Computer Science*, vol. 31, pp. 40-48, 2008.

[4] E. A. Haroutunian, I. A. Safaryan, H.M. Petrosyan and A. R. Gevorkian, "On identification of anomalies in multidimensional hydrogeochemical data as earthquake precursors", *Mathematical Problems of Computer Science,* vol. 40, pp. 76-84, 2013.

[5] E. A. Haroutunian, I. A. Safaryan, A. Nazaryan and N. Harutyunyan, "Detection of heterogeneity on three-dimensional data sequences: algorithm and applications", *Mathematical Problems of Computer Science,* vol. 42, pp. 63-72, 2014.

Application of Sanov's Theorem to Testing of Random Variables Independence

E.A. Haroutounian, A.O. Yesayan

Institute for Informatics and Automation Problems of NAS, Armenia
E-mail: *evhar@ipia.sci.am, armfrance@yahoo.fr*

Introduction. In this paper we consider the classical hypotheses testing problem of independence of two variables and more random variables with getting exponential decay for error probability.

Problem statement. Let $(\mathbf{X}, \mathbf{Y})$ be the result of N independent observations or N-samples of random variables (RVs) (X, Y). If X and Y are discrete, their possible values are $x_1, x_2, ..., x_M$ and $y_1, y_2, ..., y_L$ respectively. Let n_{ml} is the number of pairs (x_m, y_l) in N-sample, for which $X = x_m$, $Y = y_l$, $m = \overline{1, M}$, $l = \overline{1, L}$. If X and Y are continuous, the domain of their values can be presented by intervals
$[x_1, x_2), [x_2, x_3), ..., [x_m, x_{m+1}), ..., [x_{M-1}, x_M)$ and
$[y_1, y_2), [y_2, y_3), ..., [y_l, y_{l+1}), ..., [y_{L-1}, y_L)$. Then n_{ml} are frequencies of elements , for which X and Y belong respectively to m-th and l-th intervals. Based on this N sample we have to test hypotheses

$$H_0 : X \text{ and } Y \text{are independent}$$

against

$$H_1 : X \text{ and } Y \text{are dependent.}$$

If H_0 is true it follows that $p_{ml} = p_{m.} \times p_{.l}$, $m = \overline{1, M}$, $l = \overline{1, L}$ where

$$p_{m.} = \frac{n_{m.}}{N},\ p_{.l} = \frac{n_{.l}}{N},\ n_{m.} = \sum_{l=1}^{L} n_{ml},\ n_{.l} = \sum_{m=1}^{M} n_{ml}.$$

The problem of testing of these hypotheses for given first type error probability $\alpha = P(H_1/H_0)$ (wrong acceptance H_1 when H_0 is correct) can be solved using Pearson χ^2 chi-square test.
We calculate the observed value of χ^2

$$\chi^2_{obs} = \sum_{m,l} \frac{(n_{ml} - N \times p_{ml})^2}{N \times p_{ml}}$$

and define critical region by the condition $P(\chi^2 > \chi^2_{crit.}) = \alpha$. From the table of χ^2 distribution we find χ^2_{crit} (critical value of χ^2). If $\chi^2_{obs} < \chi^2_{crit}$

we accept H_0.
We get for large enough N exponential decrease of α using Sanov's theorem.
Sanov's theorem [1]. Let $\mathcal{P}$ be the set of all probability distributions and $X_1, X_2, ..., X_N$ be identically independently distributed by distribution G, $\mathcal{E} \subset \mathcal{P}$ and $\mathcal{E}$ is the closure of its interior, then

$$\lim_{N->\infty} \log \frac{1}{N} G^N(\mathcal{E}) = -D(Q^*||G)$$

where $Q^* = \arg\min_{Q\in\mathcal{E}} D(Q||G)$ is the distribution in $\mathcal{E}$ that is closest to G in relative entropy.

Suppose $\mathcal{E}$ is the set of joint distributions of RVs which are dependent. So applying Sanov's theorem we get

$$\alpha \approx \exp\{-ND(Q^*||p_{m.} \times p_{.l})\} = \exp\{-N\mathbf{I}(X \wedge Y)\},$$

$$\text{where } \{Q^* = \frac{n_{ml}}{N}, \, m = \overline{1, M}, \, l = \overline{1, L}\} \in \mathcal{E}.$$

.

In general case we have hypotheses

$$H_0 : \text{The group of } K \text{ RVs are independent}$$

against

$$H_1 : \text{The group of } K \text{ RVs are dependent.}$$

For simplifying notations we take three variables. Applying again Sanov's theorem we get $\alpha \approx \exp\{-ND(Q^*||p_{m..} \times p_{.l.} \times p_{..r})\}$ where empirical distributions are used.

Acknowledgement. This work was supported in part by SCS of MES of RA under Thematic Program No SCS 13-1A295.

References

[1] Th. M. Cover, J. A. Thomas. Elements of information theory. *John Willey and Sons inc. publication*, second edition, 2006.

Երկրորդ կարգի կորերի կիզակետերի պրոյեկտիվ որոշման մասին

Ս. Ք. ՀԱՐՈՒԹՅՈՒՆՅԱՆ

Խ. Աբովյանի անվան հայկական պետական մանկավարժական համալսարան
E-mail: *S_Haroutunian@netsys.am*

Էվկլիդեսյան հարթության մեջ էլիպսի, հիպերբոլի և պարաբոլի սահմանումներում օգտագործվում է երկու կետերի հեռավորության հասկացությունը, որը բացակայում է աֆինական և պրոյեկտիվ երկրաչափություններում: Մյուս կողմից, նույնիսկ Էվկլիդեսյան հարթության մեջ երկրորդ կարգի գծերի դասակարգումը կրում է աֆինական բնույթ: Առաջանում է երկրորդ կարգի կորերի աֆինական և այնուհետև պրոյեկտիվ սահմանման խնդիր: Սույն աշխատանքը նվիրված է այդ կորերի կիզակետերի պրոյեկտիվ նկարագրման խնդրին: Աֆինական տեսանկյունից էլիպսը, հիպերբոլը և պարաբոլը հարթության բոլոր այն կետերի բազմություններն են, որոնցից յուրաքանչյուրի համար տրված կետից (կիզակետից) և տրված ուղղից (դիրեկտրիսից) հեռավորությունների հարաբերությունը հաստատուն է (հավասար է էքսցենտրիսիտետին): Պրոյեկտիվ հարթության մեջ այդ երեք կորերն ունեն ընդհանուր հենք՝ երկրորդ կարգի ձվածիրը: Այդ կորի միջոցով սահմանվում են կորի կենտրոնի, համալուծ տրամագծերի զույգերի, առանցքների հասկացությունները: Դրանց կիզակետերի հասկացությունը ներմուծելու համար անհրաժեշտ է կապարել անհատականացում, օրինակ, ներմուծելով դիրեկտրիսը: Այդ դեպքում հնարավոր է կառուցել համապատասխան կիզակետը: Հակառակը եթե տրված է այդ կորերից որևէ մեկի կիզակետը, ապա միարժեքորեն կառուցվում է համապատասխան դիրեկտրիսը: Այդ բոլոր կառուցումները հարմար է կապարել ընդլայնված Էվկլիդեսյան կամ աֆինական հարթության մոդելում: Այդ մոդելում առանձնացվում է այսպես կոչված անվերջ հեռու կետերի ուղիղը, որի վրա առաջանում է որոշակի ինվոլյուցիա: Այդ ինվոլյուցիան կարող է ունենալ կամ չունենալ անշարժ կետեր: Դրան համապատասխան ստացվում են հիպերբոլ (երկու անշարժ կետ), պարաբոլ (մեկ անշարժ կետ) և էլիպս (անշարժ կետերը բացակայում են): Այդ դեպքերից յուրաքանչյուրի համար նկարագրվում է կիզակետի կառուցման գործընթացը:

Holomorphic Besov spaces of holomorphic functions on the polydisk and unit ball in C^n

A. Harutyunyan

Yerevan State University, Armenia
E-mail: *anahit@ysu.am*

Let U^n be the unit polydisk in C^n and S be the space of functions of regular variation. Let $1 \leq p < \infty$, $omega = (\omega_1, \dots, \omega_n)$, $\omega_j \in S(1 \leq j \leq n)$ and $f \in H(U^n)$. The function f is said to be an element of the holomorphic Besov space $B_p(\omega)$ if

$$|f|^p_{B_p(\omega)} = \int_{U^n} |Df(z)|^p \prod_{j=1}^{n} \frac{\omega_j(1-|z_j|)}{(1-|z_j|^2)^{2-p}} dm_{2n}(z) < +\infty$$

where $dm_{2n}(z)$ is the $2n$-dimensional Lebesgue measure on U^n. We show that $B_p(\omega)$ is a Banach space with respect to $||\cdot||_{B_p(\omega)}$ and the set of polynomials is dense in $B_p(\omega)$. The properties of the functions in S can be found in [1].

Next we consider ω- weighted Besov spaces of holomorphic functions on the unit ball in C^n . Let B^n be the unit ball in C n and S be the space of functions of regular variation. Let $0 < p < +\infty$, The function f is said to be in holomorphic Besov space $B_p(\omega)$ if

$$||f||^p_{B_p(\omega)} = \int_{B^n} (1-|z|^2)^p |Df(z)|^p \frac{\omega(1-|z|)}{(1-|z|2)^{n+1}} d\nu(z) < +\infty$$

where $d\nu(z)$ is the volume measure on B^n . We describe the holomorphic Besov space in terms of the corresponding L_p space. Projection theorems and theorems of existence of inverse are proved. We also give explicit descriptions of the duals of these spaces.([2])

References

[1] A.V.Harutyunyan, W.Lusky, *Weighted holomorphic Besov spaces on the polydisks,* Function Spaes and Applications, vol. 9,1(2011), 1-16

[2] A.V.Harutyunyan, W.Lusky, (2011) $\omega-$ weighted holomorphic Besov space on the unit ball in C^n, Comment. Math.Univ. Carolin 52,1 (2011) pp.37-56

The spectral theory of the family of Sturm-Liouville operators

T.N. Harutyunyan

Yerevan State University, Armenia
E-mail: *hartigr@yahoo.co.uk*

Let $\mu_n(q,\alpha,\beta)$, $n=0,1,2,\dots$, are the eigenvalues of the Sturm-Liouville problem $L(q,\alpha,\beta)$:

$$-y''+q(x)\,y=\mu y,\ x\in(0,\pi),\ q\in L_R^1[0,\pi],$$

$$y(0)\cos\alpha+y'(0)\sin\alpha=0,\ \alpha\in(0,\pi],$$

$$y(\pi)\cos\beta+y'(\pi)\sin\beta=0,\ \beta\in[0,\pi).$$

The first question that we want to answer is:

How to move the eigenvalues, when (α,β) change on $(0,\pi]\times[0,\pi)$.

For this purpose we introduce the concept of the eigenvalues function (EVF).

Definition: *The function* $\mu_q(\cdot,\cdot)$, *defined on* $(0,\infty)\times(-\infty,\pi)$ *by formula*

$$\mu_q(\alpha+\pi k,\beta-\pi m)\overset{def}{=}\mu_{k+m}(q,\alpha,\beta),\ k,m=0,1,2,\dots,$$

called the eigenvalues function (EVF) of the family of problems $\{L(q,\alpha,\beta),\ \alpha\in(0,\pi],\ \beta\in[0,\pi)\}$.

We find that this function has many properties, which we can investigate and the answer to our first question is:

When (α,β) change on $(0,\pi]\times[0,\pi)$, then the set of the eigenvalues form an analytic surface which we called EVF.

We find necessary and sufficient conditions for function of two variables having these properties to be the EVF of the family of problems $\{L(q,\alpha,\beta),\alpha\in(0,\pi],\ \beta\in[0,\pi)\}$. In particular an algorithm for solving the inverse problem is given.

Граничная задача Римана в весовых пространствах

Г.М. Айрапетян, В.Г. Петросян

Ереванский государственный университет, Армения
E-mail: *hhayrapet@gmail.com*

Пусть $\rho(t) = |t-t_1|^{\alpha_1}...|t-t_m|^{\alpha_m}, t_k \in T$, где $T = \{z, |z| = 1\}$ единичная окружность и $\alpha_k, k = 1, 2...m$ действительные числа. Через

$$\rho_r(t) = \rho^*(t)|r^{\delta_1}t - t_1|^{n_1}...|r^{\delta_m}t - t_m|^{n_m}$$

обозначим функцию где $\rho^*(t) = |t - t_1|^{\lambda_m}...|t - t_1|^{\lambda_m}$,

$$\delta_k = \begin{cases} 1, & \text{если } \alpha_k \le -1 \ , \\ 0, & \text{если } \alpha_k > -1 \ , \end{cases}$$

$$n_k = \begin{cases} [\alpha_k] + 1, & \text{если } \alpha_k \text{ нецелое}, \\ \alpha_k, & \text{если } \alpha_k \text{ целое}, \end{cases}$$

$\lambda_k = \alpha_k - n_k$. Ясно что $\lambda_k \in (-1, 0]$, и $\rho^*(t) \in L^1(T)$.
Рассмотрим граничную задачу Римана А в следущей постановке:
Задача А. Пусть f произвольная измеримая на T функция из класса $L^1(\rho)$. Определить аналитическую в $D^+ \cup D^-$, где $D^+ = \{z; |z| < 1\}$, $D^- = \{z; |z| > 1\}$ функцию $\Phi(z), \Phi(\infty) = 0$, так чтобы имело место граничное условие

$$\lim_{r \to 1-0} \|\Phi^+(rt) - a(t)\Phi^-(r^{-1}t) - f(t)\|_{L^1(\rho_r)} = 0, \qquad (1)$$

Где $a(t), a(t) \neq 0$ произвольная функция из класса $C^\delta(T)$,$\delta > 0$, $\Phi^\pm$ сужения функции Φ на $D^\pm$ соответственно. Обозначим $\kappa = inda(t), t \in T$. Аналогичная задача, когда постановке когда $\rho(t) \equiv 1$ иследована в работе [1]

В работе устанавливается, что если $\sum_{k=1}^{m} n_k + \kappa \ge 0$, то задача А разрешима для любой функции f. При $\sum_{k=1}^{m} n_k + \kappa < 0$ получены необходимые и достаточные условия разрешимости этой задачи. Решения получены в явном виде.

Список литературы

[1] Г.М. Айрапетян. Разрывная задача Римана-Привалова со смещением в классе L^1. *Изв. АН Арм. ССР мат.*, XXУ, 1, 3-20, 1990.

Short exact sequences of some subalgebras of the Toeplitz algebra

K.H. Hovsepyan

Kazan State Power Engineering University, Russia
E-mail: *karen.hovsep@gmail.com*

Introduction. In this paper we consider some subalgebras of Toeplitz algebra, for which there exist short exact sequences.

Section 1. Let T be a shift opertaor acting on a Hilbert space $l^2(\mathbb{Z}_+)$ in the following way: $Te_n = e_{n+1}$, where $\{e_n\}_{n=0}^{\infty}$ is an orthonormal basis in $l^2(\mathbb{Z}_+)$. C^*-algebra generated by the operator T is called Toeplitz algebra and is denoted by $\mathcal{T}$. It is obvious, that each element of $\mathcal{T}$ has the form T^nT^{*l} for some $n, l \in \mathbb{N}$. Element T^nT^{*l} of the Toeplitz algebra we call *monomial*, and a number $n - l$ *index* of the monomial T^nT^{*l}. Let $\mathcal{T}(m)$ be a C^*-subalgebra of the Toeplitz algebra, generated by the operators T^m, T^{*m}, and $\mathcal{T}_m$ subalgebra of the Toeplitz algebra, generated by all monomials, index of which is divisible by m. It is evident, that $\mathcal{T}(m) \subset \mathcal{T}_m$.

Lemma 1. *Let $\mathcal{K}_m$ be a subalgebra of compact operators in $\mathcal{T}_m$, then*

$$\mathcal{K}_m \cong \bigoplus^{m} \mathcal{K} = \mathcal{K} \oplus ... \oplus \mathcal{K},$$

where $\mathcal{K}$ is a subalgebra of compact operators of the Toeplitz algebra.

We denote by J_i the ideal of $\mathcal{K}_m$, i-component of which is 0, that is:

$$J_i = \mathcal{K} \oplus ... \oplus \mathcal{K} \oplus 0 \oplus \mathcal{K} \oplus ... \oplus \mathcal{K}.$$

It is evident, that $\{J_i\}_{i=1}^{m}$ is a family of maximal ideals of the algebra $\mathcal{K}_m$.

Section 2.

Definition 1. *Sequences of type* $0 \longrightarrow A \xrightarrow{q} B \xrightarrow{r} C \longrightarrow 0$ *are called short exact sequences, where q - monomorphism, r - epimorphism, and* $\ker(q) = \mathrm{im}(r)$. *If there exist *-homomorphism $t : C \to B$, such that $r \circ t = id_C$, then short exact sequence is called splittable.(see [1])*

In the work [2] Coburn proved the following result:

Lemma 2. *There exists short exact sequence:*

$$0 \to \mathcal{K} \to \mathcal{T} \to C(S^1) \to 0,$$

where $C(S^1)$ - is the algebra of all continuous functions on S^1.

We prove the following Theorems using the Lemma noted above.

Theorem 1. *There exist short exact sequences:*

$$0 \to \mathcal{K}(m) \to \mathcal{T}(m) \to C(S^1) \to 0,$$

$$0 \to \mathcal{K}_m \to \mathcal{T}_m \to C(S^1) \to 0.$$

Theorem 2. *There exist short exact splittable sequences:*

$$0 \to \bigcap_{k=1}^{n} J_{i_k} \to \mathcal{T}_m \to \mathcal{T}_n \to 0,$$

where $1 \leq i_1 < i_2 < ... < i_n \leq m$*. Moreover,* $\mathcal{T}_m$ *is isomorphic to the direct sum of algebras:*

$$\mathcal{T}_m \cong \mathcal{T}_n \oplus \bigcap_{k=1}^{n} J_{i_k}.$$

Corollary 1. *The following short exact sequences are splittable:*

$$0 \to J_i \to \mathcal{T}_m \to \mathcal{T}(m) \cong \mathcal{T} \to 0,$$

where $1 \leq i \leq m$*.*

References

[1] K.R. Davidson. C^*-algebras by example. *Fields Institute Monograph.*, 6, AMS, 1996.

[2] L.A. Coburn The C^*-algebra generated by an isometry II. *Bull. Amer. Math. Soc.*, (62) 1010–1013, 1969.

Constructive method of the factorization matrix function

A.G. Kamalyan

Yerevan State University, Armenia
E-mail: *kamalyan_armen@yahoo.com*

We will discuss one constructive method of the factorization matrix function. This method is based on structural properties of kernel of Teoplits operators.

On an equivalency of differentiation basis of dyadic rectangles

G.A. Karagulyan, D.A. Karagulyan, M.H. Safaryan

Yerevan State University, Armenia
E-mail: *g.karagulyan@yahoo.com, davidkar@yahoo.com, mher.safaryan@gmail.com*

Introduction. Let $\mathcal{R}$ be the family of half-closed rectangles $[a,b)\times[c,d)$ in $\mathbb{R}^2$. Then let $\mathcal{R}^{\text{dyadic}}$ be the family of dyadic rectangles of the form

$$\left[\frac{i-1}{2^n},\frac{i}{2^n}\right)\times\left[\frac{j-1}{2^m},\frac{j}{2^m}\right),\quad i,j,n,m\in\mathbb{Z}, \tag{1}$$

We have $\mathcal{R}^{\text{dyadic}}\subset\mathcal{R}$. For a given rectangle $R\in\mathcal{R}$ we denote by $\text{len}(R)$ the length of the bigger side of R. A family of rectangles $\mathcal{M}\subset\mathcal{R}$ is said to be a differentiation basis (or simply basis), if for any point $x\in\mathbb{R}^2$ there exists a sequence of rectangles $R_k\in\mathcal{M}$ such that $x\in R_k$, $k=1,2,\ldots$ and $\text{len}(R_k)\to 0$ as $k\to\infty$. For a differentiation basis $\mathcal{M}\subset\mathcal{R}$ and for any function $f\in L^1(\mathbb{R}^2)$ define

$$\delta_{\mathcal{M}}(x,f)=\limsup_{\substack{\text{len}(R)\to 0\\ x\in R\in\mathcal{M}}}\frac{1}{|R|}\int_R f(t)dt-\liminf_{\substack{\text{len}(R)\to 0\\ x\in R\in\mathcal{M}}}\frac{1}{|R|}\int_R f(t)dt.$$

The integral of a function $f\in L^1(\mathbb{R}^2)$ is said to be differentiable at a point $x\in\mathbb{R}^2$ with respect to the basis $\mathcal{M}$, if $\delta_{\mathcal{M}}(x,f)=0$. Consider classes of functions

$$\mathcal{F}(\mathcal{M})=\{f\in L(\mathbb{R}^2):\ \delta_{\mathcal{M}}(x,f)=0 \text{ almost everywhere }\},$$
$$\mathcal{F}^+(\mathcal{M})=\{f\in L(\mathbb{R}^2):\ f(x)\geq 0,\ \delta_{\mathcal{M}}(x,f)=0 \text{ almost everywhere }\}.$$

The following classical theorems determine the optimal Orlicz space for the functions having a.e. differentiable integrals with respect to the entire family of rectangles $\mathcal{R}$, which is the space $L(1+\log L)(\mathbb{R}^2)\subset L^1(\mathbb{R}^2)$, corresponding to the case $\Phi(t)=t(1+\log^+ t)$ ([1]).

Theorem 1 (Jessen-Marcinkiewicz-Zygmund, [2])**.** $L(1+\log L)(\mathbb{R}^2)\subset$ $\mathcal{F}(\mathcal{R})$.

Theorem 2 (Saks, [4])**.** *If* $\Phi(t)=o(t\log t)$ *as* $t\to\infty$, *then* $\Phi(L)(\mathbb{R}^2)\not\subset\mathcal{F}(\mathcal{R})$. *Moreover, there exists a positive function* $f\in\Phi(L)(\mathbb{R}^2)$ *such that* $\delta_{\mathcal{R}}(x,f)=\infty$ *everywhere.*

Such theorems are valid also for the basis $\mathcal{R}^{\text{dyadic}}$. The first one trivially follows from embedding $L(1+\log L)(\mathbb{R}^2) \subset \mathcal{F}(\mathcal{R}) \subset \mathcal{F}(\mathcal{R}^{\text{dyadic}})$. The second can be deduced from the relation $\mathcal{F}^+(\mathcal{R}^{\text{dyadic}}) = \mathcal{F}^+(\mathcal{R})$, due to Zerekidze [7].

Let $\Delta = \{\nu_k : k = 1, 2, \ldots\}$ be an increasing sequence of positive integers. This sequence generates the rare basis $\mathcal{R}^{\text{dyadic}}{}_{\Delta}$ of dyadic rectangles of the form (1) with $n, m \in \Delta$. This kind of bases first considered in the papers [6], [5]. Stokolos [6] proved that the analogous of Saks theorem holds for any basis $\mathcal{R}^{\text{dyadic}}{}_{\Delta}$ with an arbitrary Δ sequence. That means $L(1 + \log L)(\mathbb{R}^2)$ is again the largest Orlicz space containing in $\mathcal{F}(\mathcal{R}^{\text{dyadic}}{}_{\Delta})$. G. A. Karagulyan [3] proved some theorems, establishing an equivalency of some convergence conditions for multiple martingale sequences, those in particular imply some results of the papers [6], [5].

In this paper we prove

Theorem 3. *Let $\Delta = \{\nu_k\} \subset \mathbb{N}$ be an increasing sequence of positive integers. Then the condition*

$$\sup_{k \in \mathbb{N}} (\nu_{k+1} - \nu_k) < \infty$$

is necessary and sufficient for the equality $\mathcal{F}(\mathcal{R}^{\text{dyadic}}{}_{\Delta}) = \mathcal{F}(\mathcal{R}^{\text{dyadic}})$.

References

[1] M. Guzman, *Differentiation of integrals in $\mathbb{R}^n$*, Springer-Verlag (1975).

[2] B. Jessen, J. Marcinkiewicz, A. Zygmund, *Note of differentiability of multiple integrals*, Fund. Math., 25(1935), 217–237 .

[3] G. A. Karagulyan, *On equivalency of martingales and related problems*, Izvestia NAN Armenii(English translation in Journal of Contemporary Mathematical Analysis), 48(2013), No 2, 51–65.

[4] S. Saks , *Remark on the differentiability of the Lebesgue indefinite integral*, Fund. Math., 22(1934), 257–261.

[5] K. Hare and A. Stokolos, A. *On weak type inequalities for rare maximal functions*, Colloq. Math., 83 (2000), no. 2, 173–182.

[6] A. Stokolos, *On weak type inequalities for rare maximal function in $\mathbb{R}^n$*, Colloq. Math., 104(2006), no. 2, 311–315.

[7] T. Sh. Zerekidze, *Convergence of multiple Fourier-Haar series and strong differentiability of integrals*, Trudy Tbiliss. Mat. Inst. Razmadze Akad. Nauk Gruzin. SSR, 76 (1985) , 80–99.(Russian)

About on the first cohomology group for a β-uniform algebra with coefficients in $\mathbb{Z}$

M.I. Karakhanyan

Yerevan State University, Armenia
E-mail: *m_karakhanyan@yahoo.com*

Let $C_b(\Omega)$be a Banach algebra of a all bounded, complex-valued, continuous functions a locally compact, Hausdorf space Ω provided with the uniform norm. Using a family of a seminorms $\{P_g\}_{g\in C_0(\Omega)}$ one can define with the help of on ideal $C_0(\Omega)$ a topology on algebra $C_b(\Omega)$, where $P_g(f) = \|T_g f|_\infty = \sup\limits_{\Omega}|gf|$, and $T_g : C_b(\Omega) \to C_b(\Omega)$ is a multiplication operator $T_g f = gf$. In this topology an algebra $C_b(\Omega)$ is called a β-uniform topology and is defined as $C_\beta(\Omega)$ ([1]-[3]).

Let $\mathcal{F}$ be a filtering system in $C_0(\Omega)$, for which a family of seminorms $\{P_F\}_{F\in\mathcal{F}(\Omega)}$ is defined a β-uniform topology in algebra $C_\beta(\Omega)$.

Let $\mathcal{A}(\Omega)$ is a β-uniform subalgrbra in algebra $C_\beta(\Omega)$ (see [4]). We suppose that the family of seminorms $\{P_F\}_{F\in\mathcal{F}(\Omega)}$ gives a β-uniform topology of algebra $\mathcal{A}(\Omega)$.

We denote by $\mathcal{A}_F(\Omega)$ a completion by a norm $\|\cdot\|_{\infty,F} = {P_F(\cdot)}\Big/{Ker(P_F)}$ of an algebra ${\mathcal{A}(\Omega)}\Big/{Ker(P_F)}$ for each $F\in\mathcal{F}(\Omega)$ be a commutative Banach algebra.

Let π_F be an algebraic morphism $\mathcal{A}(\Omega)\to\mathcal{A}_F(\Omega)$ which is a superposition of a canonical epimorphism $\tau_F = \mathcal{A}(\Omega) \to {\mathcal{A}(\Omega)}\Big/{Ker(P_F)}$ and the natural injection $J_F : {\mathcal{A}(\Omega)}\Big/{Ker(P_F)} \to \mathcal{A}_F(\Omega)$ that is $\pi_F = J_F\circ\tau_f$. Because an algebra $\mathcal{A}(\Omega)$ is a β-uniform. The algebra $\mathcal{A}(\Omega)$ coincides (to with in isomorphism) with the proectiv limits of the system of Banach algebras $(\mathcal{A}_F(\Omega);\pi_{F,H})$, that is $\mathcal{A}(\Omega) = \lim\limits_{\leftarrow}(\mathcal{A}_\beta(\Omega);\pi_{F,H})$ (see [5]).

Let $\mathcal{A}^{-1}(\Omega)$ be a group of inverse elements, for a β-uniform algebra $\mathcal{A}(\Omega)$. Then the group $\mathcal{A}^{-1}(\Omega)$ is an open set of a β-uniform algebra $\mathcal{A}(\Omega)$, because a some p-sphere (ball) with a unit origin is contained in $\mathcal{A}^{-1}(\Omega)$ (see [6]). Let $\mathcal{A}^{(1)}(\Omega)$ be a main component of a group $\mathcal{A}^{-1}(\Omega)$.

Theorem 1. *The following diagram is a commutative diagram of an in-*

ective homorphismes

$$\begin{array}{ccc} \mathcal{A}^{-1}(\Omega) \Big/ \mathcal{A}^{(1)}(\Omega) & \longrightarrow & \lim\limits_{\leftarrow} \mathcal{A}^{-1}(\Omega) \Big/ \exp(\mathcal{A}_F(\Omega)) \\ \downarrow & & \downarrow \\ C_\beta(\Omega)^{-1} \Big/ C_\beta^{(1)}(\Omega) & \longrightarrow & \lim\limits_{\leftarrow} C^{-1}(M(\mathcal{A}_F(\Omega))) \Big/ \exp(C(M(\mathcal{A}_F(\Omega)))) \end{array}$$

and

$$\lim_{\leftarrow} \mathcal{A}_F^{-1}(\Omega) \Big/ \exp(\mathcal{A}_F(\Omega)) \xrightarrow{\sim} \lim_{\leftarrow} H^1\left(M(\mathcal{A}_F(\Omega)), \mathbb{Z}\right).$$

The proof follows from a classical Arens-Roiden theorem for commutative Banach algebras and the facts, that for a β-uniform algebras there take place

$$\mathcal{A}(\Omega) = \lim_{\leftarrow} \left(\mathcal{A}_F(\Omega); \pi_{FH}\right) \text{ and } C_\beta(\Omega) = \lim_{\leftarrow} \left(C_F(\Omega); \pi_{FH}\right).$$

As a corollary the following results hold.

Theorem 2. *Let $\mathfrak{b}\Omega = M(C_b(\Omega))$ be the space of a maximal ideals of a Banach algebra $C_\beta(\Omega)$ where $\mathfrak{b}\Omega$ is a Stoun-Chekh compactification for Ω. Then the group $C_\beta^{-1}(\Omega) \Big/ \exp(C_\beta(\Omega))$ is isomorphic to the image of homomorphism $H^1(\mathfrak{b}\Omega, \mathbb{Z}) \to H^1(\Omega, \mathbb{Z})$.*

References

[1] R. Buck. Bounded continuous functions on a locally compact space. *Michigem Math. J.*, 5:95-104, 1958.

[2] R. Giles. A generalization of the strict topology. *Trans. of the Amer. Math. Soc.*, 161:467-474, 1971.

[3] M. Karakhanyan, T. Khor'kova. A characterization of the algebra $C_\beta(\Omega)$. *Funct. Anal. and Applic.*, 13(1):69-71, 2009.

[4] S. Grigoryan, M. Karakhanyan, T. Khor'kova. On β-uniform dirichlet algebras. *Izv. Nats. Acad. Nauk Armenii, Math.*, 45(6):17-26, 2010.

[5] A. Robertson, W. Robertson. Topological vector spaces. *Cambridge Univ. Press*, 1964.

[6] W. Rudin. Functional Analysis. *New York, Toronto.*, 1973.

Integral representation through the differential operator and embedding theorems for multianisotropic spaces

G.A. Karapetyan

Russian-Armenian Slavonic University, Armenia
E-mail: *Garnik_Karapetyan@yahoo.com*

Let R^n is n-dimensional space, Z_+^n is the set of multi-indices. For the set of multi-indices let's denote through $\aleph$ the smallest convex polyhedron, containing all the points of the given set. The polyhedron is said to be a completely correct, if:

a) has a vertex in the origin of coordinates and in all the coordinate axes;

b) outward normal's of all $(n-1)$ -dimensional non-coordinate faces are positive.

Let μ_i is the outward normal of the face $\aleph_i^{(n-1)}$ such, that $\forall\alpha \in \aleph_i^{(n-1)} (\alpha,\mu^i) = \alpha_1\mu_1^i + \cdots + \alpha_n\mu_n^i = 1; \left|\mu^i\right| = \left|\mu_1^i\right| + \cdots + \left|\mu_n^i\right|$. Let us denote through $W_p^{\aleph}(R^n)$ the set of all measurable functions in R^n for which $f \in L_p(R^n)$ and for any $\forall\alpha^i \in \aleph_i^{(n-1)} D^{\alpha_i} f \in L_p(R^n), i = 1,\cdots,M$.

In the present work the integral representation through the differential operator is offered, which is generated via the polyhedron $\aleph$ and applying the obtained integral representation, the embedding of the set $W_p^{\aleph}(R^n)$ in $L_q(R^n)$ is proved.

Theorem 1. *Let $\aleph$ is a convex polyhedron and $f \in L_p(R^n)$ and $\forall\alpha \in \partial'\aleph$ $D^{\alpha_1} f \in L_p(R^n)$. Let the multi-index β and the numbers $1 \le p \le q \le \infty$ are such, that $(\beta;\mu) + \left(\frac{1}{p} - \frac{1}{q}\right)|\mu| < 1$, for any normal μ of the $(n-1)$ -dimensional hyper-plane of the polyhedron $\aleph$.*

Let

$$\max(\beta;\mu) + \left(\frac{1}{\mathrm{p}} - \frac{1}{\mathrm{q}}\right)|\mu| = (\beta;\mu_0) + \left(\frac{1}{\mathrm{p}} - \frac{1}{\mathrm{q}}\right)|\mu_0|.$$

Then $D^\beta W_p^{\aleph}(R^n)$ embedding $L_q(R^n)$, i.e. for any $f \in W_P^{\aleph}(R^n)$ there exists $D^\beta f \in L_q(R^n)$ and the following estimation is true

$$\|D^\alpha f\|_{L_q(R^n)} \le C_1 h^{1-\left((\beta;\mu_0)+\left(\frac{1}{p}-\frac{1}{q}\right)|\mu_0|\right)} \sum_{i=1}^{M} \|D^\alpha f\|_{L_p(R^n)} +$$
$$+ C_2 h^{-\left((\beta;\mu_0)+\left(\frac{1}{p}-\frac{1}{q}\right)|\mu_0|\right)} \|f\|_{L_p(R^n)},$$

where C_1, C_2 *are numbers independent of* f, h, *and* h *is a parameter, which varies in* $0 < h < h_0$.

On One Integral Equation with Chebyshev Polynomial Nonlinearity

A.Kh. Khachatryan, Kh.A. Khachatryan, Ts.E. Terdjyan

Institute of Mathematics of NAS, Armenia,
Armenian National Agrarian University, Armenia
E-mail: *Aghavard@hotbox.ru, Khach82@rambler.ru, Terjyan73@mail.ru*

We consider an integral equation on half-line with Chebyshev polynomial nonlinearity, arising in dynamic theory of universe and p-adic string theory.

We prove existence of the positive and monotonically increasing continuous solution in class of essentially bounded functions on half-line. We also found two sided estimates for obtained solution, as well as the limit of solution at infinity.

We prove uniqueness of a solution in the certain class of functions.

We generalize the results for more general integral equation with "double" nonlinearity.

At the end we give some examples of functions, describing nonlinearity.Using suggested constructive solution method we present some results of numerical calculations, having direct application in cosmology.

Acknowledgement. This work was supported by state of Science MES RA in frame of the research project SCS 13YR-1A0003.

The Interior Transmission Eigenvalue Problem for a Spherically-Symmetric Domain with Anisotropic Medium and a Cavity

A. Kirsch, H. Asatryan

Karlsruhe Institute of Technology, Germany
Yerevan State University, Armenia
E-mail: *andreas.kirsch@kit.edu, asthaik@ysu.am*

We consider the scattering of spherically-symmetric acoustic waves by an anisotropic medium and a cavity. While there is a large number of recent works devoted to the scattering problems with cavities, existence of an infinite set of transmission eigenvalues is an open problem in general. We prove existence of an infinite set of transmission eigenvalues for anisotropic Helmholtz equation in a spherically-symmetric domain with a cavity. Further we consider the corresponding inverse problem. Under some assumptions we prove the uniqueness in the inverse problem.

Acknowledgement. The research of H. Asatryan has been supported by the German Academic Exchange Service (DAAD).

Симметрические уравнения в свободном моноиде с параметрическими показателями

А.Ш. Малхасян

Ереванский государственный университет, Армения
E-mail: *amalkhasyan@mail.ru*

В 1977 году [1] Г.С.Маканин впервые доказал, что существует алгоритм, распознающий разрешимость произвольной системы уравнений в свободном моноиде. Несмотря на многочисленные усилия, до сих пор остается открытой проблема описания общего решения уравнения в свободном моноиде. Известны некоторые методы таких описаний; параметризация, описания с помощью графов, описания с помощью функций, определенных в работах Г.С.Маканина (см.[2]), и ряд других. В упомянутой работе [2] Г.С.Маканиным было найдено общее решение симметрического уравнения

$$x_1x_2\dots x_{n-1}x_n = x_nx_{n-1}\dots x_2x_1 \tag{1}$$

в свободном моноиде с алфавитом образующих

$$a_1, a_2, \dots, a_m \tag{2}$$

В настоящем докладе сообщается что получено описание общего решения уравнения вида

$$x_1^{\lambda_1}x_2^{\lambda_2}\dots x_{n-1}^{\lambda_{n-1}}x_n^{\lambda_n} = x_n^{\lambda_n}x_{n-1}^{\lambda_{n-1}}\dots x_2^{\lambda_2}x_1^{\lambda_1}, \tag{3}$$

где λ_j параметры, принимающие целые неотрицательные значения.

Список литературы

[1] Г.С. Маканин. Проблема разрешимости уравнений в свободной полугруппе. *Матем. сб.*, 103(145), 2(6), 147-236, 1977.

[2] Г.С. Маканин. Параметризация решений уравнения $x_1x_2\dots x_{n-1}x_n = x_nx_{n-1}\dots x_2x_1$ в свободном моноиде. *Матем. заметки*, 89; 6,879-884, 2011.

Approximation of functions using the scaled Laplace transform

R.M. Mnatsakanov, K. Sarkisian

West Virginia University, USA
National Institute for Occupational Safety and Health, USA
E-mail: *rmnatsak@stat.wvu.edu, vtq0@cdc.gov*

Introduction. In this talk the problem of recovering a function F, its derivative, and the primitive function given the Laplace transform of the underlying function F is studied.

Recall that the problem of inverting the Laplace transform represents very severe ill-posed inverse problem. We refer to [1] were the rate of convergence of regularized version of the inverse of the Laplace transform was studied. In [2]-[3] the moment-recovered approximations of a cumulative distribution function (cdf) F and its probability density function (pdf) $f = F'$ were suggested and their asymptotic properties were investigated. Let us mention only two different methods for approximation of the Laplace transform inversion, see [4] and [5] among others. The former article uses the maximum entropy method, while in the later one the moment-recovered approach that is based on the several moments of F.

In our talk we present the uniform upper bounds for the approximation errors and demonstrate their asymptotic behavior via the plots and tables.

Section 1. Let us suppose that the cdf F is absolute continuous with respect to the Lebesgue measure and has a support in $\mathbb{R}_+ = [0, \infty)$. Denote by f the corresponding density function of F with respect to the Lebesgue measure on $\mathbb{R}_+$. Given the values of the Laplace transform of F

$$\mathcal{L}_F(s) = \int_{\mathbb{R}_+} e^{-s\,\tau}\, dF(\tau), \quad \text{as} \quad s \in \{0, \ln b, 2\ln b, \ldots, \alpha \ln b\},$$

we suggest the approximations of Laplace transform inversions recovering F and f, respectively:

$$F_{\alpha,b}(x) = 1 - \sum_{k=0}^{[\alpha b^{-x}]} \sum_{j=k}^{\alpha} \binom{\alpha}{j} \binom{j}{k} (-1)^{j-k}\, \mathcal{L}_F(j \ln b)$$

$$f_{\alpha,b}(x) = \frac{[\alpha b^{-x}]\,\Gamma(\alpha+2)}{\alpha\,\Gamma([\alpha b^{-x}]+1)} \sum_{m=0}^{\alpha-[\alpha b^{-x}]} \frac{(-1)^m\, \mathcal{L}_F((m+[\alpha b^{-x}])\ln b)}{m!\,(\alpha-[\alpha b^{-x}]-m)!}\,, \qquad (1)$$

as $x \in \mathbb{R}_+$ and $\alpha \to \infty$ (cf. with [5] and [6]). We assume that the scaling parameter $b \in (1, \exp(1))$. The problem of choosing the optimal value of b by minimizing the approximation error will be addressed as well.

Similar questions in the multivariate case will be discussed. In particular, using the two-dimensional version of (1), we approximate the probability density function f as follows:

$$f_{a,b}(x,y) = \frac{[\alpha b^{-x}][\alpha' b^{-y}] \ln^2(b)\, \Gamma(\alpha+2)\, \Gamma(\alpha'+2)}{\alpha\, \alpha' \Gamma([\alpha b^{-x}]+1)\, \Gamma([\alpha' b^{-y}]+1)}$$
$$\times \sum_{m=0}^{\alpha-[\alpha b^{-x}]} \sum_{l=0}^{\alpha'-[\alpha' b^{-y}]} \frac{(-1)^m \mathcal{L}_F((m+[\alpha b^{-x}]) \ln b,\, (l+[\alpha' b^{-y}]) \ln b)}{m!\, (\alpha-[\alpha b^{-x}]-m)!\, l!\, (\alpha'-[\alpha' b^{-y}]-l)!},$$

where $x, y \in \mathbb{R}_+$ and $a = (\alpha, \alpha') \in \mathbb{N} \times \mathbb{N}, \mathbb{N} = \{1, 2, \ldots\}$.

References

[1] D.E. Chauveau, A.C.M. van Rooij, F.H. Ruymgaart, Regularized inversion of noisy Laplace transforms, *Advances in Appl. Math.*, 15: 186-201, 1994.

[2] R.M. Mnatsakanov, Hausdorff moment problem: Reconstruction of distributions, *Statist. Probab. Letters*, 78: 1612-1218, 2008.

[3] R.M. Mnatsakanov, Hausdorff moment problem: Reconstruction of probability density functions, *Statist. Probab. Letters*, 78: 1869-1877, 2008.

[4] A. Tagliani, Y. Velasquez, Numerical inversion of the Laplace transform via fractional moments, *J. of Appl. Math. and Comput.*, 143: 99-107, 2003.

[5] R.M. Mnatsakanov, Moment-recovered approximations of multivariate distributions: The Laplace transform inversion, *Statist. Probab. Letters*, 81: 1-7, 2011.

[6] R.M. Mnatsakanov, K. Sarkisian, A note on recovering the distributions from exponential moments. *J. of Appl. Math. and Comput.*, 219: 8730–8737, 2013.

Non-idempotent Plonka Functions and weakly Plonka Sums

Yu.M. Movsisyan, D.S. Davidova

Yerevan State University, Armenia
European Regional Academy, Armenia
E-mail: *yurimovsisyan@yahoo.com, di.davidova@yandex.ru*

There exist various extensions of the concept of a lattice. In this talk we study weakly idempotent lattices with an additional interlaced operation. We characterize interlacity of a weakly idempotent semilattice operation, using the concept of hyperidentity; and prove that a weakly idempotent bilattice with an interlaced operation is epimorphic to the superproduct with negation of two equal lattices. In the last part of the talk we introduce the concepts of a non-idempotent Plonka function and the weakly Plonka sum and extend the main result for algebras with the well known Plonka function to the algebras with the non-idempotent Plonka function. As a consequence we characterize the hyperidentities of the variety of weakly idempotent lattices, using non-idempotent Plonka functions, weakly Plonka sums and characterization of cardinality of the sets of operations of subdirectly irreducible algebras with hyperidentities of the variety of weakly idempotent lattices. Applications of weakly idempotent bilattices in multi-valued logic is to appear.

Boolean-Linear Quasigroups

Yu.M. Movsisyan, G. Rustamyan

Yerevan State University, Armenia
E-mail: *yurimovsisyan@yahoo.com, gor.rustamyan@gmail.com*

The present paper is devoted to the study of a special type of quasigroups of the order 2^n, which we call boolean-linear quasigroups.

Let $(Q, *)$ be a quasigroup of the order 2^n and β be a bijective mapping $\beta : Q \to \mathbb{Z}_2^n$, where $\mathbb{Z}_2^n$ is the n-ary boolean vector field. Then there exists a uniquely defined mapping $f : \mathbb{Z}_2^n \times \mathbb{Z}_2^n \to \mathbb{Z}_2^n$ for which the following equality is valid:

$$f(x_1, \ldots, x_n, y_1, \ldots, y_n) = \beta(\beta^{-1}(x_1, \ldots, x_n) * \beta^{-1}(y_1, \ldots, y_n)),$$

or in a more compact form: $x * y = \beta^{-1}(f(\beta(x), \beta(y)))$. We omit β and write $x * y = f(x, y)$ in the text below.

Definition 1. *The quasigroup $(Q, *)$ is called a boolean-linear quasigroup (or, in short, a BL-quasigroup), if it can be represented in the following form:*

$$x * y = f(x, y) = A_1(x) \cdot y + b_1(x) = A_2(y) \cdot x + b_2(y),$$

where $A_1(x)$, $A_2(y)$ are $n \times n$ matrices, and $b_1(x)$, $b_2(y)$ are $n \times 1$ matrices over $\{0, 1\}$.

Theorem 1. *If quasigroup $(Q, *)$ is a BL-quasigroup, then*
1. $\forall x \in Q$, $det(A_1(x)) = det(A_2(x)) = 1$.
If matrices A_1, A_2 and b_1, b_2 satisfy the following identities:
2. $\forall x \in Z_2^n$, $det(A_1(x)) = det(A_2(x)) = 1$,
3. $\forall x, y \in Z_2^n$, $A_1(x) \cdot y + b_1(x) = A_2(y) \cdot x + b_2(y)$,
then Z_2^n with the operation defined by the rule: $x * y = f(x, y) = A_1(x) \cdot y + b_1(x)$ forms a BL-quasigroup.

Theorem 2. *If $(Q, *)$ is a BL-quasigroup, then $A_1(x) = A_2(x)$ iff $b_1(x) = b_2(x)$.*

Theorem 3. *The BL-quasigroup $(Q, *)$ is a commutative iff $A_1(x) = A_2(x), \forall x \in Q$.*

Theorem 4. *A BL-quasigroup $(Q, *)$ has an identity element, or, in other words, is a loop, iff there exists $e \in Q$ such that $b_1(e) = b_2(e) = 0$ and $A_1(e) = A_2(e) = E$.*

Theorem 5. *If BL-quasigroup is a loop, then it is commutative.*

Let $(Q, *)$ be BL-loop. It follows from the previous theorems that $A_1(x) = A_2(x) = A(x)$ and $b_1(x) = b_2(x) = b(x)$. We make the following denotations: $A(0) = A_0$ and $A_0 \cdot e = b_0$.

Theorem 6. *If $(Q, *)$ is a BL-loop, then $x * y = A(x) \cdot y + A_0 \cdot x + b_0 = A(y) \cdot x + A_0 \cdot y + b_0$.*

Definition 2. *A Moufang loop is the loop Q that satisfies the following equivalent identities:*

1. $x(y * xz) = (xy * x)z$,
2. $(zx * y)x = z(x * yx)$,
3. $(xy)(zx) = (x * yz)x$,
4. $(xy)(zx) = x(yz * x)$.

These identities are known as Moufang identities. If a Moufang loop is commutative (i.e. the identity $x * y = y * x$ holds in the loop), then the loop satisfies the following identity:

5. $x^2 * yz = xy * z$.

Theorem 7. *The BL-loop is a Moufang loop iff it satisfies the following identities:*

1. $A(A_0 \cdot x) \cdot A(x) \cdot y + A(A_0 \cdot y) \cdot A(x) \cdot x + A(A_0 \cdot x) \cdot A_0 \cdot x + A(A_0 \cdot y) \cdot A_0 \cdot x + A(A_0 \cdot x) \cdot b_0 + A(A_0 \cdot y) \cdot b_0 + A(x * y) \cdot b_0 + A(x * x) \cdot b_0 + A_0 \cdot A_0 \cdot x + A_0 \cdot A_0 \cdot y = 0$,

2. $A(x * y) \cdot A(x) = A(x * x) \cdot A(y)$.

In the case of $e = 0$, the first identity can be simplified:

1. $A^2(x) \cdot y + (A(y) \cdot A(x) + A(x) + A(y)) \cdot x + (x + y) = 0$.

References

[1] H. Pflugfelder, *Quasigroups and Loops: Introduction*, Helderman Verlag Berlin, 1990.

[2] D. Gligoroski, S. Markovski, and S. J. Knapskog, *Public Key Block Cipher Based on Multivariate Quadratic Quasigroups.*, Report 320, Cryptology ePrint Archive, 2008.

[3] V. Belousov, Moufang loops. *in Hazewinkel Michiel, Encyclopedia of Mathematics, Springer*, 2001.

[4] Yu. Movsisyan, Introduction to the theory of algebras with hyperidentities. *Yerevan State University Press*, Yerevan, 1986. (Russian)

The relationship between covariograms of a cylinder and its base

V.K. Ohanyan

Yerevan State University, Armenia
E-mail: *victo@aua.am*

Let $\mathbb{R}^n$ be the $n-$dimensional Euclidean space, $D \subset \mathbb{R}^n$ be a bounded convex body, S^{n-1} be the $(n-1)$-dimensional unit sphere centered at the origin and $L_n(\cdot)$ be the n-dimensional Lebesgue measure in $\mathbb{R}^n$.
G. Matheron formulated a hypothesis that in the class of all bounded convex bodies, a bounded convex body is determined by its covariogram. This hypothesis is known as Matheron's conjecture (see [1]). In [2], G. Bianchi and G. Averkov confirmed Matheron's conjecture for $n = 2$. G. Bianchi has also proved that for $n \geq 4$ the hypothesis is false. Very little is known regarding the covariogram problem when the space dimension is greater than 2. It is known that centrally symmetric convex bodies in any dimension, are determined by their covariogram up to translations. For $n = 3$ the problem is open. Nevertheless, for the case of bounded convex polyhedron for n=3 Matheron's conjecture received a positive answer (see [3], [1]). Thus, investigation of the covariogram of three dimensional convex bodies becomes an important first step in the study of Matheron's conjecture in $\mathbb{R}^3$. Note that the explicit form for the covariogram of three dimensional convex bodies is known only in the case of a ball. The function $C_D(x) = L_n(D \cap \{D + x\})$ $x \in \mathbb{R}^n$, is called the covariogram of the body D. Let $\mathbf{G}$ be the space of all lines in the Euclidean plane $\mathbb{R}^2$, $g \in \mathbf{G}$ and (p, φ) are the polar coordinates of the foot of the perpendicular to g from the origin, $p \geq 0$, $\varphi \in S^1$. For a closed bounded convex domain $D \subset \mathbb{R}^2$ we denote by $S_D(\varphi)$ the support function in direction $\varphi \in S^1$ defined by $S_D(\varphi) = \max\{p \in \mathbb{R}^+ : \ g(p, \varphi) \cap D \neq \emptyset\}$, where $\mathbb{R}^+$ is the set of nonnegative real numbers. For a bounded convex domain $D \subset \mathbb{R}^2$ we denote by $b_D(\varphi)$ the breadth function in direction $\varphi \in S^1$, that is, the distance between two support lines to the boundary of D that are perpendicular to φ. We have $b_D(\varphi) := S_D(\varphi) + S_D(\varphi + \pi)$. For a bounded convex domain D the chord length distribution function in direction φ, denoted by $F_D(x, \varphi)$, is defined to be the probability of having chord $\chi(g) = g \cap D$ with length at most x in the bundle of lines parallel to φ. A random line which is parallel to φ and intersects D has an intersection point (denoted by y) with the line l_φ. The intersection point y is uniformly distributed on the segment $[0, b_D(\varphi)]$. Thus we have $F_D(x, \varphi) = \frac{L_1\{y : \chi(l_\varphi + y) \leq x\}}{b_D(\varphi)}$. The

orientation dependent chord length distribution function and the covariogram for $n = 2$ are known only in the cases of a disc, a triangle, a regular polygon, a parallelogram and an ellipse (see [4]–[6]). Denote by Γ the space of lines γ in $\mathbb{R}^3$. Let $\Pi_D(\omega)$ denote the projection of a bounded convex body $D \subset \mathbb{R}^3$ in direction $\omega \in S^2$ and let $s_D(\omega)$ be its area. Every line which is parallel to ω and intersects D has an intersection with $\Pi_D(\omega)$. Denote that point by y and that line by $l_\omega + y$. The intersection point y is uniformly distributed on $\Pi_D(\omega)$. The chord length distribution function of D in direction $\omega \in S^2$ is defined by $F_D(x,\omega) = \frac{L_2\{y : \chi(l_\omega + y) \leq x\}}{s_D(\omega)}$. In the paper [7] the following results are obtained: (1) Relationships between the covariogram and the orientation-dependent chord length distribution function of a cylinder and those of its base. (2) Explicit forms of the covariogram and the orientation-dependent chord length distribution function of a cylinder with cyclic, elliptical and triangular bases.

References

[1] R. Schneider, W. Weil, Stochastic and Integral Geometry *Springer, Berlin-Heidelberg*, 2008.

[2] G. Bianchi, and G. Averkov, Confirmation of Matheron's Conjecture on the covariogram of a planar convex body' *Journal of the European Mathematical Society*, 11: 1187–1202, 2009.

[3] Bianchi G., The covariogram determines three-dimensional convex polytopes, *Adv. Math.*, 220: 1771-1808, 2009.

[4] A. G. Gasparyan, V. K. Ohanyan, Recognition of triangles by covariogram, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 48 (3): 110–122, 2013.

[5] H. S. Harutyunyan, V. K. Ohanyan, Orientation-dependent sections distributions for convex bodies, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 49 (3): 139–156, 2014.

[6] A. G. Gasparyan, V. K. Ohanyan, Covariogram of a parallelogram, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 49 (4): 194–206, 2014.

[7] H. S. Harutyunyan and V. K. Ohanyan, The covariogram of a cylinder, *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)*, 49 (6): 366–375, 2014.

Касательные векторные поля на гиперповерхностях евклидовых пространств

А.А. Огникян

Ереванский государственный университет, Армения

Гиперповерхность $S^{p,q}$ в евклидовом пространстве размерности $n = p + q + 1$ определяется как пространство, образованное всеми точками $x = (x_0, x_1, \ldots, x_{p+q})$, где $x_0^2 + x_1^2 + \ldots + x_p^2 - x_{p+1}^2 - \ldots - x_{p+q}^2 = 1$ Рассматривается задача нахождения прямоугольных образующих у этих гиперповерхностей, то есть требуется описать пространство всех прямых, целиком лежащих на данной гиперповерхности. Например, хорошо известно, что гиперповерхность $S^{1,1}$ (однополостный гиперболоид) является дважды линейчатой поверхностью.

Решение общей задачи тесно связано с задачей построения касательных линейно независимых векторных полей на $S^{p,q}$. Например, гиперповерхность $S^{1,2}$ параллелезуема, и наличие на ней в явном виде трех касательных линейно независимых векторных полей позволяет доказать, что пространство прямолинейных образующих для $S^{1,2}$ гомеоморфно 2-мерному тору. Более того, становится возможным составлять в явном виде уравнения всех прямолинейных образующих, проходящих через данную точку гиперповерхности.

Основываясь на конструкции из [1], для любого i, $q \geq i \geq 0$ на гиперповерхности $S^{p,q}$ в явном виде строятся $\rho(p+i) + q - i$ линейно независимых касательных векторных полей, где ρ - функция Радона-Гурвица. Обозначим $r(p,q) = max(\rho(p+i) + -i)$, где $q \geq i \geq 0$, и пусть $R(p,q)$ максимальное число касательных линейно независимых векторных полей на $S^{p,q}$. Следовательно, имеет место

Теорема 1. $R(p,q) \geq r(p,q)$.

Список литературы

[1] А.А. Огникян. Комбинаторное построение векторных полей на сферах. *Мат. заметки*, 83(4):590-605, 2008.

On the constructive solution of an inverse Sturm-Liouville problem

A.A. Pahlevanyan

Institute of Mathematics of National Academy of Sciences, Armenia
E-mail: *apahlevanyan@instmath.sci.am*

Let $L(q,\alpha,\beta)$ be the Sturm-Liouville boundary value problem

$$-y'' + q(x)\,y = \mu y \equiv \lambda^2 y,\ x \in (0,\pi),\ \mu \in \mathbb{C},$$

$$y(0)\cos\alpha + y'(0)\sin\alpha = 0,\ \alpha \in (0,\pi],$$

$$y(\pi)\cos\beta + y'(\pi)\sin\beta = 0,\ \beta \in [0,\pi),$$

where q is a real-valued, summable function on $[0,\pi]$ $\left(q \in L^1_{\mathbb{R}}[0,\pi]\right)$.

The question **"What kind must be the sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$, for them to be the spectrum and the norming constants of a problem $L(q,\alpha,\beta)$, respectively"** is well studied for the cases $q \in L^2_{\mathbb{R}}[0,\pi]$, $\alpha,\beta \in (0,\pi)$ and $\alpha=\pi$, $\beta=0$ ([1, 2, 3, 4, 5, 6, 7]). For the case $q \in L^1_{\mathbb{R}}[0,\pi]$ and $\sin\alpha = 0$ $(\alpha=\pi)$, $\beta \in (0,\pi)$ (analogously for $\alpha \in (0,\pi)$, $\sin\beta = 0$ $(\beta = 0)$) some aspects of this question have also been studied by the above mentioned and other authors, but, to our knowledge there are no necessary and sufficient conditions for the sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ to be the spectrum and the norming constants for the problem $L(q,\pi,\beta)$ (analogously for $L(q,\alpha,0)$). We solve this problem in the following theorem:

Theorem 1. *For the two sequences $\{\mu_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=0}^{\infty}$ to be the spectrum and the norming constants of a problem $L(q,\pi,\beta)$, with $q \in L^1_{\mathbb{R}}[0,\pi]$ and $\beta \in (0,\pi)$, it is necessary and sufficient that the following relations hold:*

$$\sqrt{\mu_n} \equiv \lambda_n = n + \delta_n(\pi,\beta) + \frac{c}{2(n+\delta_n(\pi,\beta))} + l_n + O\left(\frac{1}{n^2}\right),$$

$$a_n = \frac{\pi}{2(n+\delta_n(\pi,\beta))^2}\,(1+s_n),$$

where $\delta_n(\alpha,\beta)$ *is the solution of the following transcendental equation*

$$\delta_n(\alpha,\beta) = \frac{1}{\pi}\arccos\frac{\cos\alpha}{\sqrt{(n+\delta_n(\alpha,\beta))^2\sin^2\alpha+\cos^2\alpha}} - \frac{1}{\pi}\arccos\frac{\cos\beta}{\sqrt{(n+\delta_n(\alpha,\beta))^2\sin^2\beta+\cos^2\beta}},$$

c *is a constant, the reminders* $l_n = o\left(\frac{1}{n}\right)$, $s_n = o\left(\frac{1}{n}\right)$ *and they are such that the functions*

$$l(x,\beta) = \sum_{n=2}^{\infty} l_n \sin(n+\delta_n(\pi,\beta))x,$$

$$s(x,\beta) = \sum_{n=2}^{\infty} s_n \cos(n+\delta_n(\pi,\beta))x$$

are absolutely continuous on arbitrary segment $[a,b] \subset (0,2\pi)$, *uniformly with respect to* $\beta \in (0,\pi)$ *and* q *from the bounded subsets of* $L^1_{\mathbb{R}}[0,\pi]$.

References

[1] Marchenko, V.A. “Some questions of the theory of one-dimensional linear differential operators of the second order.” *Trudy Moskov. Mat. Obsh.*, 1, (1952): 327–420.

[2] Gelfand, I. M. and Levitan, B.M. “On the determination of a differential equation from its spectral function.” *Izv. Akad. Nauk SSSR, ser. Math.*, 15, no. 4, (1951): 309–360.

[3] Gasymov, M.G. and Levitan, B.M. “Determination of a differential equation by two of its spectra.” *UMN*, 19, no. 2, (1964): 3–63.

[4] Zhikov, V.V. “On inverse Sturm-Liouville problems on a finite segment.” *Izv. Akad. Nauk SSSR, ser. Math.*, 31, no. 5, (1967): 965–976.

[5] Isaacson, E.L. and Trubowitz, E. “The inverse Sturm-Liouville problem. I.” *Comm. Pure Appl. Math.*, 36, no. 6, (1983): 767–783.

[6] Pöschel, J. and Trubowitz, E. *Inverse spectral theory*, Academic Press, Inc., Boston, MA, 1987.

[7] Freiling, G. and Yurko, V.A. *Inverse Sturm-Liouville Problem and Their Applications*, NOVA Science Publishers, New York, 2001.

Reciprocity Laws and Arithmetic Geometry

M. Papikian

Pennsylvania State University, USA
E-mail: *papikian@psu.edu*

Introduction. The simplest example of a reciprocity law is Gauss's Quadratic Reciprocity. I will give an overview of generalizations of quadratic reciprocity studied in modern number theory. Then I will describe such reciprocity laws arising from elliptic curves and Drinfeld modules. Finally, I will discuss how to derive from a particular reciprocity law a criterion for the splitting modulo primes of a class of non-solvable polynomials over $\mathbb{F}_q(T)$ studied by Abhyankar. (This is a joint work with Alina Cojocaru.)

References

[1] A. Cojocaru, M. Papikian. Drinfeld modules, Frobenius endomorphisms, and CM-liftings. *International Mathematics Research Notices*, to appear.

Duality in weight spaces of functions harmonic in the unit ball

A.I. Petrosyan

Yerevan State University, Armenia
E-mail: *apetrosyan@ysu.am*

A positive continuous decreasing function φ on $[0,1)$ is called *weight function* if $\lim \varphi(r) = 0$, as $r \to 1$, and a positive finite Borel measure η on $[0,1)$ is called *weighting measure* if it is not supported in any subinterval $[0,\rho)$, $0 < \rho < 1$. Let $h_\infty(\varphi)$ be the Banach space of functions u, harmonic in the unit ball $B_n \subset \mathbb{R}^n$, with the norm $\|u\|_\varphi = \sup\{|u(x)|\varphi(|x|) \colon\ x \in B_n\}$ and let $h_0(\varphi)$ be the closed subspace of functions u with $|u(x)| = o\big(1/\varphi(|x|\big)$ as $|x| \to 1$.

It has been shown by Rubel and Shields, [1] that $h_\infty(\varphi)$ is isometrically isomorphic to the second dual of $h_0(\varphi)$. In [2], in the case $n = 2$, it was posed and solved the duality problem of finding a weighting measure η such that

$$h^1(\eta) = \big\{v \in L^1(d\eta(r)\, d\sigma) \colon\ v \text{ is harmonic in } B_2\big\}$$

represents the intermediate space, the dual of $h_0(\varphi)$ and the predual of $h_\infty(\varphi)$, i.e. $h^1(\eta) \sim h_0(\varphi)^*$ and $h^1(\eta)^* \sim h_\infty(\eta)$.

It is well-known that in the case $n = 2$ every harmonic function h has expansion in a series on degrees z and $\overline{z}$ in unit disk $|z| < 1$, since real-valued harmonic function is a real part of a holomorphic function.

We consider duality problem in the case of harmonic functions in the unit ball of $\mathbb{R}^n$, $n > 2$. The multidimensional case has the specifics in the sense that we can not speak about connection between harmonic and holomorphic functions, and instead of degrees z and $\overline{z}$ we deal with spherical harmonics.

We use the same approach to the duality problem as [2]. This approach depends on showing (see [3]) that a certain integral operator from $L^\infty(d\eta(r)\, d\sigma)$ to $h_\infty(\eta)$ is a bounded projection. The kernel of the integral operator is the reproducing kernel for $h_\infty(\eta)$ (see [5] for details).

One could work with the analogous spaces $A_0(\varphi)$ and $A_\infty(\varphi)$ of functions which are holomorphic in the unit ball of $\mathbb{C}^n$, and study the analogous duality problem. It is shown in [4] that this duality problem is solvable if φ is *normal*.

The essential part of the definition of *normal* is that $1/\varphi(r)$ grows slower than some power of $1/(1-r)$ but faster than some other power.

In the report we suppose, that weight function grows more slowly than some power of $1/(1-r)$. Thus, we have a solution to the duality problem for *non-normal* weight functions as

$$\varphi(r) = \left(\ln \frac{e}{1-r}\right)^{-\alpha}, \quad \alpha > 0$$

and

$$\varphi(r) = \left(\ln \ln \frac{e^e}{1-r}\right)^{-\alpha}, \quad \alpha > 0.$$

References

[1] L.A. Rubel, A.L. Shields, *The second duals of certain spaces of analytic functions.* J. Austral. Math. Soc, **11**: 276–280, 1970.

[2] A.L. Shields, D.L. Williams, *Bounded projections, duality, and multipliers in spaces of harmonic functions.* J. Reine Angew. Math. **299–300**: 256–279, 1978.

[3] A.I. Petrosyan, E.S. Mkrtchyan. *Duality in spaces of functions harmonic in the unit ball.* Proceedings of the Yerevan State University, no. 3, 28–35, 2013.

[4] A.I. Petrosyan, *Bounded Projectors in Spaces of Functions Holomorphic in the Unit Ball.* Journal of Contemporary Mathematical Analysis, **46**(5): 264–272, 2011.

[5] A.I. Petrosyan, *On weighted classes of harmonic functions in the unit ball of* $\mathbf{R}^n$. Complex Variables, **50**(12): 953–966, 2005.

Higher regularity of the free boundary in the elliptic Signorini problem

Arshak Petrosyan

Purdue University, USA
E-mail: *arshak@math.purdue.edu*

One of the classical approaches in the proof of the higher regularity of free boundaries is the hodograph-Legendre transform. A generalization of this approach to the Signorini problem, where the free boundary is thin (i.e. has co-dimension two) leads to a singular hodograph transform which can be shown to be invertible by using a precise asymptotic behavior of the solutions. The corresponding Legendre transform solves a fully nonlinear degenerate elliptic equation, which surprisingly has a subelliptic structure. Treating it as an appropriate perturbation of the Baouendi-Grushin operator, we are able to prove the smoothness and even the real analyticity of the Legendre transform, which in turn implies the real analyticity of the free boundary.

This is joint work with Herbert Koch and Wenhui Shi.

The Modeling of the Priority Problem with Some Extensions of Petri Nets

G.R. Petrosyan

Armenian State Pedagogical University after Kh. Abovyan, Armenia
E-mail: *petrosyan_gohar@list.ru*

In this talk the problem of modeling of a priority process is presented [1, page 189]. The problem is discussed for several extensions of Petri Net traits. A modeling of Petri Net traits is built, which describes the mentioned process both in the presence of Restrictive Arc Petri Net and Colored Petri Net. The comparison of complexity in advanced nets is carried out. The problem is discussed for an optimization process having comparison of nets.

Introduction. A Petri Net (also known as a place/transition net or P/T net) is one of several mathematical modeling languages for description of distributed systems. A Petri Net is a directed bipartite graph, in which the nodes represent transitions (i.e. events that may occur, are signified by bars) and places (i.e. conditions, signified by circles). The directed arcs describe the places which are pre/post conditions for transitions (signified by arrows). A Petri Net consists of *places*, *transitions*, and *arcs*. Graphically, the places in a Petri Net can contain a discrete number of marks called *tokens*. Any distribution of tokens over the places represent a configuration of the net called a *marking*. In an abstract sense, relating to a Petri Net diagram, a transition of a Petri Net can *fire* if it is *enabled*, i.e. there are sufficient tokens in all of its input places; when the transition fires, it consumes the required input tokens, and creates tokens in its output places. Unless an execution *policy* is defined, the execution of Petri Nets is nondeterministic: if the multiple transitions are enabled at the same time, any one of them can fire. Since firing is nondeterministic, and as multiple tokens can exist anywhere in the net (even in the same place), Petri Nets are well suited for modeling the concurrent behavior of distributed systems [1]-[2].
The Restrictive Arc Petri Net is quintuplets: $C = (P_1, P_2, T, I, O)$. P_1- finite set of basic positions, P_2- finite set of restrictive positions, T - finite set of transitions, where $P_1 \cap P_2 = \emptyset$, $P_1 \cap T = \emptyset$, $P_2 \cap T = \emptyset$. Denote‘ $P = P_1 \cup P_2$. $I : T \to P^\infty$, $O : T \to P^\infty$ are the input and output functions, respectively, where P^∞ are all possible collections (repetitive elements) of P.

Definition: A Colored Petri Net is a tuple $CPN = (\sum, P, T, A, N, C, G, E, I)$ satisfying the following requirements:
(i) $\sum$ is a finite set of non-empty types, called **color sets**.
(ii) P is a finite set of **places**.
(iii) T is a finite set of **transitions**.
(iv) A is a finite set of **arcs** such that:
$P \cap T = P \cap A = T \cap A = \emptyset$
(v) N is a **node** function. It is defined from A into $P \times T \cup T \times P$.
(vi) C is a **color** function. It is defined from P into $\sum$.
(vii) G is a **guard** function. It is defined from T resulting the expressions such that: $\forall t \in T : [Type(G(t)) = Bool \wedge Type(Var(G(t))) \subseteq \sum]$.
(viii) E is an **arc expressions**function. It is defined from A resulting the expressions such that:
$\forall a \in A : [Type(E(a)) = C(p(a))MS \wedge Type(Var(E(a))) \subseteq \sum]$, where $p(a)$ is the place of $N(a)$.
(ix)I is an **initialization** function. It is defined from P resulting the closed expressions such as: $\forall p \in P : [Type(I(p)) = C(p)_{MS}]$.
Colored Petri Net is a graphical oriented language, which is used for modeling, analysis, description and presentation systems [3].
In the classical or traditional Petri Net, tokens do not differ from each other, and we can say that they are colorless. In contrast to Classical Petri Nets, the position of Colored Petri Nets can contain tokens of arbitrary complexity - a note, lists, etc., which makes the reliable models more possible. Let us assume, there are two processes of producers and there are two processes of consumers. The producers should collect the data in the buffer and the consumers should coordinate them for activities in usage of the channel [1]. The idea of priority does not let the mentioned system be modeled by the Classical Petri Net. The proof of the following fact is shown in details in [[1]page 190-191]. For solving the mentioned problem we extend several traits of Petri Net in such a way, which are headed to opportunity of zero checking in Petri Net.

References

[1] J. Peterson. Petri Net Theory and the Modeling of Systems. *Prentice Hall. ISBN 0-13-661983-5*, 1981.

[2] T. Murata Petri nets: Properties, Analysis and Applications. *Proc. of the IEEE*, 74(4), 1989.

[3] K. Jensen Coloured Petri Nets: Basic Concepts, Analysis Methods and Practical Use. *Basic Concepts. Monographs in Theoretical Computer Science, Springer - Verlag, Berlin, Germany,* V1,V2,V3, 1997.

Շոշափող շերտավորման ներքին կապակցության մասին

Վ.Ա. ՓԻԼԻՊՈՍՅԱՆ

Երևանի պետական համալսարան
E-mail: *vpiliposyan@ysu.am*

Դիցուք M-ը n-չափանի դիֆերենցելի բազմաձևություն է, T_xM-ը՝ x կետում նրա շոշափող փարածությունն է, իսկ $TM = \{(x;v), x \in M, v \in T_xM\}$-ը M-ի շոշափող շերտավորումն է, որում ներմուծված է

$$(\mathrm{x}^{\alpha}) = (x^i; x^{\bar{i}}) = (x^1; x^2; ...; x^n; v^1; v^2; ...; v^n)$$

լոկալ կորդինատային համակարգ, որտեղ x^i-երը բազային, իսկ $x^{\bar{i}}$-երը շերտի կորդինատներն են: Ինչպես հայտնի է, կամայական $\mathrm{x} \in T_xM$-ի համար $\Delta_v(\mathrm{x}) = \{(0;v) \in T_{\mathrm{x}}(TM)\}$-ը $T_{\mathrm{x}}(TM)$-ի ենթափարածություն է, որը ինվարիանտ է կորդինատային ձևափոխության նկատմամբ, և որը իզոմորֆ է T_xM-ին:

Դիֆերենցելի $\mathrm{x} \mapsto \Delta_h(\mathrm{x})$ բաշխումը TM-ում կոչվում է ներքին $(infinitezimal)$ կապակցություն, եթե այն ինվարիանտ է կորդինատային ձևափոխության նկատմամբ և

$$T_{\mathrm{x}}(TM) = \Delta_h(\mathrm{x}) \bigoplus \Delta_v(\mathrm{x}) :$$

Դիցուք $(\partial_i, \partial_{\bar{i}})$-ը բնական ռեպեր է, իսկ $e_i = \partial_i - \Gamma_i^{\bar{k}} \partial_{\bar{k}}, e_{\bar{i}} = \partial_{\bar{i}}$-ը այդ ներքին կապակցության համակցված ռեպեր է: Այս դեպքում, կորդինատային համակարգի ձևափոխության ժամանակ ունենք՝

$$(e'_i, e'_{\bar{i}}) = (e_i, e_{\bar{i}}) \begin{pmatrix} f_1 & 0 \\ f_3 & f_2 \end{pmatrix},$$

ընդ որում $\mathrm{x} \mapsto \Delta_h(\mathrm{x})$ բաշխումը ինվարիանտ է կորդինատային ձևափոխության նկատմամբ այն և միայն այն դեպքում, երբ

$$\Gamma' = (f_2\Gamma - f_3) f_1^{-1}) :$$

Դիցուք $\Delta : (X(TM) \times X(TM)) \to X(TM)$-ը գծային կապակցություն է շոշափող շերտավորման վրա և ω-ն նրա գծային ձևն է: Նշված գծային կապակցությունը կոչվում է լիովին բերվող, եթե հարմարեցված ռեպերում

$$\omega = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_4 \end{pmatrix}$$

Թեորեմ 1. *Որպեսզի շոշափող շերտավորման վրա տրված լիովին բերվող գծային կապակցությունը պահպանվի հարմարեցված ռեպերի ձևափոխության դեպքում անհրաժեշտ է և բավարար, որ ներքին կա– պակցության դեֆորմացիայի $T = \Gamma' - \Gamma$ տենզորը լինի կովարիանտ հաստատուն:*

Դիցուք $\Phi : TM \to T\widetilde{M}$-ը շոշափող շերտավորումների դիֆեոմորֆիզմ է, որը ինչպես հայտնի է, տրվում է

$$\widetilde{x}^i = \widetilde{x}^i(x^1, ..., x^n), \quad \widetilde{x}^{\tilde{i}} = a^i_j(x)\widetilde{x}^{\tilde{j}}, \quad det(a^i_j) \neq 0$$

հավասարումներով:

Թեորեմ 2. $\Phi : TM \to T\widetilde{M}$ *արտապատկերումը* $(x^i; x^{\tilde{i}})$ *մակածված կորդինատային համակարգը արտապատկերում է* $(\widetilde{x}^i;\widetilde{x}^{\tilde{i}})$ *մակածված կորդինատային համակարգին այն և միայն այն դեպքում, երբ նրա* Φ_* *դիֆերենցիալ արտապատկերումը բավարարում է* $\Phi_* \circ J = \widetilde{J} \circ \Phi_*$ *պայմանին, որտեղ* $J = \begin{pmatrix} 0 & 0 \\ E & 0 \end{pmatrix} = \widetilde{J}$ *համապատասխան շոշափող շերտավորումների աֆինորներն են, իսկ* $(\Phi^{\alpha}_{*\beta}) = \begin{pmatrix} \partial_j \widetilde{x}^i & 0 \\ \partial_k a^i_j & a^i_j \end{pmatrix}$:

Характеристика унитарных операторов в некоторых классах целых функции

С.Г. Рафаелян

Ереванский государственный университет, Армения
E-mail: *rafayelyans@ysu.am*

Пусть $p > 1$ и $W \in A_p$ -вес Макенхаупта, т.е $w(x) \geq 0$ измеримая на R функция и удовлетворяющая условию

$$\left(\int_{\daleth} w(x)\,\mathrm{d}x\right)\left(\int_{\daleth} w(x)^{-\frac{1}{p-1}}\,\mathrm{d}x\right) \leq C|\daleth|^p$$

где $\daleth \subset R$ произвольный интервал, $|\daleth|$-его длина, а C не зависящая от $\daleth$ константа.

Обозначим через $W^p_\sigma(w\,\mathrm{d}x)(\sigma > 0)$ пространство целых функций f экспоненциального типа с нормой

$$\int_R |f(x)|^p w(x)\,\mathrm{d}x = \|f\|^p < +\infty.$$

Для класса $W^p_\sigma(w\,dx)$ справедливы следующие утверждения:

Теорема 1. *Для $f \in W^p_\sigma(w(x)\,\mathrm{d}x)$ имеет место тождество*

$$f(z) = \int_R f(t)\frac{\sin\sigma(t-z)}{\sigma(t-z)}\,\mathrm{d}t,\ z \in C.$$

Теорема 2. *Для любого унитарного оператора U в пространстве $W^2_\pi(w(x)\,\mathrm{d}x)$ существует целая функция $K(z,\zeta)$, которая по z и по ζ из класса $W^2_\pi(\tilde{w}\,\mathrm{d}x)\,(\tilde{w} = w^{-1})$ и кроме того, для $g = Uf$ имеет место представления:*

$$g(z) = \int_R f(x)\overline{K(x,z)}\,\mathrm{d}x, \tag{1}$$

$$f(z) = \int_R g(x)K(x,z)\,\mathrm{d}x. \tag{2}$$

Кроме того, функция $K(z,\zeta)$ удовлетворяет условиям

$$a) K(z,\zeta) = K(\zeta,z),$$

$$b) \int_R K(x,\xi)K(x,\eta)\,\mathrm{d}x = \frac{\sin\pi(\xi-\eta)}{\pi(\xi-\eta)}.$$

Обратно, если ядро $K(z,\zeta)$ удовлетворяет условиям $a)$ и $b)$, то оно порождает, согласно формулам (1) и (2), унитарный операторов в пространстве $W^2_\pi(w\,\mathrm{d}x)$ и ему обратной.

Суммируемое решение одного нелинейного интегрального уравнения типа Гаммерштейна-Вольтерра на полуоси

Т.Г. Сардарян

Институт математики НАН Армении
E-mail: *Sardaryan.tigran@gmail.com*

Рассматривается следующее нелинейное интегральное уравнение типа Гаммерштейна-Вольтерра:

$$f(x) = \int\limits_{x}^{\infty} V(x,t)H(t,f(t))dt, \quad x \in R^{+} \equiv [0,+\infty), \tag{1}$$

относительно искомой вещественной и измеримой функции $f(x)$, определенной на R^+.

В уравнении (1) $V(x,t)$ - определенная на $R^+ \times R^+$ измеримая функция, допускающая следующее представление:

$$V(x,t) = \theta(t-x)\int_a^b \alpha(t,s)e^{-\alpha(t,s)(t-x)}d\sigma(s), \tag{2}$$

где $\alpha(t,s)$ - определенная на множестве $R^+ \times [a,b)$ $(0 \le a < b \le +\infty)$, измеримая функция, причем

$$\inf_{(t,s)\in R^+\times[a,b)} \alpha(t,s) \equiv \beta > 0, \tag{3}$$

$$\sup_{t\in R^+} \alpha(t,s) \equiv \alpha_0(s) < +\infty. \tag{4}$$

В (2) $\sigma(s)$ - монотонно неубывающая функция на $[a,b)$, причем

$$\sigma(b) - \sigma(a) = \int_a^b d\sigma(s) = 1, \tag{5}$$

а $\theta(\tau)$ - функция Хевисайда.

Рассмотрим следующую характерическую функцию определенную на R^+ :

$$\chi(p) = \beta c\int_a^b \frac{1}{\alpha_0(s)+p}d\sigma(s),\ p \in [0,+\infty),$$

где

$$c = 2\left(\int_a^b \frac{\beta}{\alpha_0(s)} d\sigma(s)\right)^{-1}. \tag{6}$$

Учитывая (3), (4) и (5), нетрудно убедится, что существует единственное $p_0 \in (0, +\infty)$, такое что

$$\chi(p_0) = 1. \tag{7}$$

Зафиксируем p_0.

Теорема 1. Пусть в уравнении (1) ядро $V(x,t)$ задается согласно формуле (2). Пусть, далее, существует суммируемая на R^+ функция $\beta(t)$:

$$\beta(t) \geq ce^{-p_0 t}, \quad t \in R^+$$

такая, что для функции $H(t,u)$ имеют место следующие условия:

a.
$$H(t,u) \leq u + \beta(t), \ u \geq e^{-p_0 t}, \ t \in R^+$$

$$H(t, e^{-p_0 t}) \geq ce^{-p_0 t}, \quad t \in R^+,$$

где числа c и p_0 определяются согласно (6) и (7) соответственно,

b. функция $H(t,u)$ при каждом фиксированном $t \in R^+$ монотонно возрастает по u на $[e^{-p_0 t}, +\infty)$,

c. $H(t,u)$ удовлетворяет условию Каратеодории по аргументу u на множестве $R^+ \times R^+$.

Тогда уравнение (1) имеет положительное суммируемое на R^+ решение.

Исследование выполнено при финансовой поддержке ГКН МОН РА в рамках научного проекта SCS 13-1A068.

Nonlinear Model Reduction for Complex Systems using Sparse Optimal Sensor Locations from Learned Nonlinear Libraries

S. Sargsyan, S.L. Brunton, J.N. Kutz

University of Washington, USA
E-mail: *ssusie@u.washington.edu*

We demonstrate the synthesis of sparse sampling and machine learning to characterize and model complex, nonlinear dynamical systems over a range of bifurcation parameters. First, we construct modal libraries using the classical proper orthogonal decomposition to uncover dominant low-rank coherent structures. Here, nonlinear libraries are also constructed in order to take advantage of the discrete empirical interpolation method and projection that allows for the approximation of nonlinear terms in a low-dimensional way. The selected sampling points are shown to be nearly optimal sensing locations for characterizing the underlying dynamics, stability, and bifurcations of complex systems. The use of empirical interpolation points and sparse representation facilitate a family of local reduced-order models for each physical regime, rather than a higher-order global model, which has the benefit of physical interpretability of energy transfer between coherent structures. In particular, the discrete interpolation points and nonlinear modal libraries are used for sparse representation to classify the dynamic bifurcation regime in the complex Ginzburg-Landau equation. It is shown that nonlinear point measurements are more effective than linear measurements when sensor noise is present.

Адиабатический предел в уравнениях математической физики

А.Г. Сергеев

Математический институт им. В.А.Стеклова, Россия
E-mail: *sergeev@mi.ras.ru*

Рассматриваются два вида уравнений, возникающих в математической физике — это уравнения Гинзбурга–Ландау из теории сверхпроводимости, и уравнения Зайберга–Виттена из квантовой теории поля.

Уравнения Гинзбурга–Ландау возникают в физической модели, управляемой лагранжианом Гинзбурга–Ландау. Указанный лагранжиан имеет следующий вид

$$\mathcal{L}(A,\Phi) = |F_A|^2 + |d_A\Phi|^2 + \frac{\lambda}{4}(1-|\Phi|^2)^2,$$

где A обозначает электромагнитный вектор-потенциал, задающийся 1-формой $A = A_1dx_1 + A_2dx_2$ на $\mathbb{R}^2_{(x_1,x_2)}$ с гладкими чисто мнимыми коэффициентами. Внешняя производная этой формы, имеющая вид $F_A = dA = \sum_{i,j=1}^2 F_{ij}dx_i \wedge dx_j$ с коэффициентами $F_{ij} = \partial_i A_j - \partial_j A_i$, где $\partial_j = \partial/\partial x_j$, отвечает напряженности электромагнитного поля, так что член $|F_A|^2$ совпадает с лагранжианом Максвелла. Переменная Φ задается гладкой комплекснозначной функцией $\Phi = \Phi_1 + i\Phi_2$ на $\mathbb{R}^2_{(x_1,x_2)}$, называемой иначе полем Хиггса. Физически, она описывает сверхпроводящее скалярное поле, взаимодействующее с электромагнитным полем. Ответственным за это взаимодействие является член

$$d_A\Phi = d\Phi + A\Phi = \sum_{i=1}^{2}(\partial_i + A_i)\Phi\, dx_i.$$

Функция Φ может иметь нули на плоскости, но на бесконечности $|\Phi| \to 1$. Главная специфика лагранжиана Ландау–Гинзбурга определяется членом
$\frac{1}{4}\,(1-|\Phi|^2)^2$, описывающим нелинейное "самодействие" скалярного поля Φ.

Укажем вначале, каким образом ведет себя указанная физическая система в статическом случае. Ее потенциальная энергия задается интегралом от лагранжиана по плоскости $\mathbb{R}^2_{(x_1,x_2)}$, которую удобно отождествить с комплексной плоскостью $\mathbb{C}$ с координатой $z = x_1 + ix_2$.

Решения модели, минимизирующие потенциальную энергию, называются вихрями. Согласно теореме Таубса, по любому конечному набору точек (с кратностями) на комплексной плоскости можно построить вихревое решение (A, Φ), для которого функция Φ имеет нули только в заданных точках и с заданной кратностью. Таким образом, пространство вихревых решений с заданным вихревым числом N (равным числу нулей Φ с учетом кратности) с точностью до естественной (т.н. калибровочной) эквивалентности можно отождествить с множеством наборов из N точек (с учетом кратности) на комплексной плоскости. Сопоставляя каждому такому набору комплексный полином N-й степени (со старшим коэффициентом единица), имеющий нули в заданных точках, мы видим, что указанное множество отождествляется с N-мерным комплексным пространством $\mathbb{C}^N$.

В отличие от статической задачи, допускающей, как мы видим, полное решение, рассчитывать на что-либо подобное в динамической задаче не приходится. Можно однако получить ее приближенные решения, исходя из следующей идеи, принадлежащей Мэнтону. Каждое решение динамической задачи представляет собой кривую $(A(t), \Phi(t))$ в пространстве пар (A, Φ), заданных с точностью до упомянутой выше калибровочной эквивалентности. Удобно представлять себе указанное пространство пар в виде "оврага", дно которого совпадает с пространством статических решений (так что каждая точка на дне есть статическое решение), а решение динамической задачи — в виде траектории шарика, катающегося по стенкам оврага. Если уменьшать скорость шарика, то его траектория будет прижиматься к дну оврага и в пределе превратится в точку на дне. Однако, если ввести на каждой траектории "медленное время", замедляя течение времени параллельно с замедлением шарика, то в пределе таких "масштабированных" траекторий получим уже не точку, а кривую, лежащую на дне оврага, которая является геодезической пространства статических решений относительно метрики, задаваемой кинетической энергией системы. Конечно, такие кривые не могут быть решениями исходной динамической задачи, поскольку каждая точка на такой кривой является статическим решением. Однако они приближенно описывают динамические решения, обладающие малой кинетической энергией. Нахождение указанных кривых сводится к решению уравнения Эйлера для геодезических на пространстве статических решений, наделенном кинетической метрикой. Поведение таких геодезических позволяет получить вполне конкретные выводы о поведении реальных вихрей. Например, можно показать, что два вихря, движущиеся навстречу друг другу по прямой, соединяющей их центры, после лобового соударения разлетятся под прямым углом по отношению к линии движения.

Вихревые уравнения, как выяснилось в последнее время, тесно связаны с уравнениями Зайберга–Виттена, вызвавшими настоящий бум в топологии гладких 4-мерных многообразий. Эти уравнения не инвариантны относительно изменения масштаба, поэтому для того, чтобы извлечь из них "полезную информацию" вводится параметр масштаба r, который затем устремляется к бесконечности. Таубс показал, что решение уравнений Зайберга–Виттена на 4-мерном симплектическом многообразии, задаваемое парой комплексных функций, будет вести себя при этом следующим образом. Одна из функций стремится к тождественному нулю, а нули другой аппроксимируют некоторую цепь, составленную из псевдоголоморфных кривых C_k с кратностями m_k (псевдоголоморфность понимается относительно комплексной структуры, совместимой с симплектической структурой многообразия). Одновременно исходное уравнение Зайберга–Виттена редуцируется к семейству вихревых уравнений, заданных в комплексных плоскостях, нормальных к кривым C_k. Обратно, для того, чтобы можно было восстановить решение уравнений Зайберга–Виттена по цепи $\sum m_k C_k$, семейство вихревых решений в нормальных плоскостях должно удовлетворять нелинейному $\bar{\partial}$-уравнению, которое можно рассматривать как комплексный аналог уравнения Эйлера для геодезических с "комплексным временем" .

Degenerate nonselfadjoint high-order ordinary differential equations on an infinite interval

L. Tepoyan, S. Zschorn

Yerevan State University, Armenia
E-mail: *tepoyan@yahoo.com, sveta1985@inbox.ru*

We consider the Dirichlet problem for degenerate ordinary differential equations of the form

$$Lu \equiv (-1)^m (t^\alpha u^{(m)})^{(m)} + a(-1)^{m-1}(t^{\alpha-1}u^{(m)})^{(m-1)} + pt^\beta u = f, \quad (1)$$

where $m \in \mathbb{N}, t \in (1, +\infty), \alpha \neq 1, 3, \dots, 2m-1$, $\beta \leq \alpha - 2m$, a and p are real constants and $f \in L_{2,-\beta}(1, +\infty)$. Let $\dot{W}^m_\alpha(1, +\infty)$ be the completion of $\dot{C}^m := \{u \in C^m[1, +\infty), u^{(k)}(1) = u^{(k)}(+\infty) = 0, k = 0, 1, \dots, m-1\}$ in the norm $\|u\|^2_{\dot{W}^m_\alpha} = \int_1^\infty t^\alpha |u^{(m)}(t)|^2\, dt$. For $\beta \leq \alpha - 2m$ there is a continuous embedding $\dot{W}^m_\alpha(1, +\infty) \hookrightarrow L_{2,\beta}(1, +\infty)$, which is compact for $\beta < \alpha - 2m$.

A function $u \in \dot{W}^m_\alpha(1, +\infty)$ is called generalized solution of Dirichlet problem for (1) if for every $v \in \dot{W}^m_\alpha(1, +\infty)$ holds the equality

$$(t^\alpha u^{(m)}, v^{(m)}) + a(t^{\alpha-1}u^{(m)}, v^{(m-1)}) + p(t^\beta u, v) = (f, v).$$

Let $d(m, \alpha) = 4^{-m}(\alpha-1)^2(\alpha-3)^2 \cdots (\alpha-(2m-1))^2$. Then for $u \in \dot{W}^m_\alpha(1, +\infty)$ we have exact inequality

$$\int_1^{+\infty} t^{\alpha-2m}|u(t)|^2\, dt \leq d(m, \alpha) \int_1^{+\infty} t^\alpha |u^{(m)}(t)|^2\, dt.$$

Theorem 1. *Let the following condition be fulfilled*

$$a(\alpha-1) > 0, d(m, \alpha) + \frac{a}{2}(\alpha-1)d(m-1, \alpha-2) + p > 0 \quad (2)$$

Then equation (1) *has a unique solution for every* $f \in L_{2,-\beta}(1, +\infty)$.

We have an operator $L : L_{2,\beta}(1, +\infty) \to L_{2,-\beta}(1, +\infty)$. Denote $\mathbb{L} := t^{-\beta}L, D(\mathbb{L}) = D(L)$. The operator $\mathbb{L}^{-1}$ acts in the space $L_{2,\beta}(1, +\infty)$, is continuous for $\beta \leq \alpha - 2m$ and compact for $\beta < \alpha - 2m$.

Now consider conjugate to (1) equation

$$Sv \equiv (-1)^m (t^\alpha v^{(m)})^{(m)} - a(-1)^{m-1}(t^{\alpha-1}v^{(m-1)})^{(m)} + pt^\beta v = g. \quad (3)$$

We say that $v \in L_{2,\beta}(1,+\infty)$ is a solution of (3) for $g \in L_{2,-\beta}(1,+\infty)$ if for every $u \in D(L)$ holds the equality $(Lu, v) = (u, g)$. Observe, that if the condition (2) is fulfilled then the generalized solution of the equation (3) exists and is unique for every $g \in L_{2,-\beta}(1,+\infty)$. Notice also that $\mathbb{L}^* = \mathbb{S}$, where $\mathbb{S} := t^{-\beta} S, D(\mathbb{S}) = D(S)$.

Proposition 1. *The spectra of operators* $\mathbb{L}$ *and* $\mathbb{S}$ *lie on the right half-space.*

Consider the selfadjoint differential equation

$$Lu \equiv (-1)^m (t^\alpha u^{(m)})^{(m)} + pt^{-2m} u = f, f \in L_{2,2m}(1,+\infty), \alpha \geq 0. \quad (4)$$

Theorem 2. *The domain of definition of the operator* L *consists of functions* $u \in \dot{W}^m_\alpha(1,+\infty)$*, for which the value* $u^{(m-1)}(+\infty)$ *is finite for* $\frac{1}{2} < \alpha < 1$*, and for* $2m-2k-2 < \alpha < 2m-2k-1$*,* $k = 0, 1, \ldots, m-2$*, the values* $u^{(k)}(+\infty)$ *also are finite.*

Also, notice that the values $u^{(k)}(+\infty)$, $k = 0, 1, \ldots, m-1$ cannot be given arbitrarily, they are defined by the right-hand side of the equation (4) (see [1]).

References

[1] A.A. Dezin, "Degenerate operator equations", Math. USSR Sbornik, vol. **43**, no. 3, 287 – 298 (1982).

[2] L.P. Tepoyan, "Degenerate differential-operator equations on infinite intervals", Journal of Mathematical Sciences, vol. **189**, no. 1, 164 – 172 (2013).

[3] S. Zschorn, "Nonselfadjoint degenerate differential operator equations of higher order on infinite interval", Proceedings of the YSU, Physical and Mathematical Sciences, no. 2, 39 – 45 (2014).

[4] L. Tepoyan, S. Zschorn, "Degenerate nonselfadjoint high-order ordinary differential equations on an infinite interval", Journal of Contemporary Mathematical Analysis, vol. **50**, no. 3, 107 – 113 (2015).

Gauge Theory in the Studies of the Economic Dynamics

L.G. Badalian, V.F. Krivorotov

University of London, United Kingdom
E-mail: *4112lucy@gmail.com*

Introduction

1. Gauges and how to use them in and outside of physics.

- **Gauges** are the basic measuring functions used generically for observation purposes.
- **The main idea** - a variable is presented as a parametric vector in the base space of e_i
$$Y = Y^i e_i \tag{1}$$
- **The Gaussian gauge** is among popular examples used in the pattern recognition
$$e^{\sum \frac{(x_i^0 - x_i)^2}{2}} \tag{2}$$

In the most generic sense, it represents a non-linear function of distance.

2. Gauges were first developed in Physics:

- **As settings** for focusing equations to assure adequate group of transformations.
- **Aiming** to focus on the signal by filtering out the white noise. In pattern recognition this contributes to separability - the ability to distinguish processed images as true positives while avoiding mixing them with false positives.
- **Compensation for nonlinearity**, assuring linearization and invariant measurement. Technically, invariance of vector in a gauge basis equals to finding the extremum for a functional.
$$Y = Y^i e_i \tag{3}$$
since
$$\delta Y = 0 \tag{4}$$
which is the necessary condition for equilibrium.

The equilibrium means a mutual compensation of all extant flows at a given point as the precondition for the very existence of an invariant system of measurement. The latter is necessary both for experiment reproducibility and the very ability to measure.

3. Repurposing gauges for the economics.

- **Conditions for gauging.** Let's assume that the vector of supply Y can be gauged through base demand e_i
- **The equilibrium hypothesis.** (4) is the mathematical representation of the general equilibrium, achievable by varying prices. This is the main issue of the classical economics and the bone of contention between warring econ. schools.
- **Conditions for clearing** all the flows at each and every point of sale can be derived from (4) and rewritten as

$$\frac{\delta Y^i}{Y^i} = -\frac{\delta e_i}{e_i}, \qquad Y^i e_i = max. \tag{5}$$

This means that any increase in supply must be compensated by a corresponding increase in demand by varying prices at the point of sale [1].

From Theoretical considerations to Real Equations

Assuming $x = x(t)$, $x = \{x^0, x^1, \ldots, x^N\}$, $x^0 = t$, $0 \leq i < N$ the extremum for the functional (3) can be written as $Y^k = Y^k(x)$, $e_k = e_k(x)$, $0 < k \leq K$. Differentiation brings us to $d\left(Y^k e_k\right) = 0$ and we get

$$\left(\frac{\partial Y^k}{\partial x^j} e_k + \frac{\partial e_k}{\partial x^j} Y^k\right) \dot{x}^j = \left(\frac{\partial Y^k}{\partial x^j} e_k + \frac{\partial e_i}{\partial x^j} Y^i\right) \dot{x}^j = \\ = \left(\frac{\partial Y^k}{\partial x^j} + \Gamma^k_{ij} Y^i\right) e_k \dot{x}^j. \tag{6}$$

Assuming the independence of base vectors $e_k \dot{x}^j$ we get the equations of parallel transport [5, 262]

$$\left(\frac{\partial Y^k}{\partial x^j} + \Gamma^k_{ij} Y^i\right) = 0, \qquad \left(\dot{Y}^k + \Gamma^k_{ij} \dot{x}^j Y^i\right) = 0, \tag{7}$$

where, by definition,

$$\Gamma^k_{ij} e_k = \frac{\partial e_i}{\partial x^j}, \qquad \Gamma^m_{ij} e_m e^k = \frac{\partial e_i}{\partial x^j} e^k, \qquad \Gamma^k_{ij} = \frac{\partial e_i}{\partial x^j} e^k \tag{8}$$

are the connections or Christoffel's symbols [5, 256]

Gauges - popular systems and simple examples

Exponential functions $e^{\pm\varphi}$, where $\varphi(x,t)$ is the cumulate, x - is the growth factor, t - time.

Exponents $e_i = e^{\pm\varphi_i}$, $\varphi_i(x,t) = \int \nu_i(x,\dot{x},t)\,dt$.

The base equations are produced by gauging $e_i = e^{-\varphi_i}$, $\varphi_i = \int \nu_i(x,\dot{x},t)\,dt$, considering $\frac{\partial e_i}{\partial x^j} = -\frac{\partial\varphi_k}{\partial x^j}\delta_i^k e_k$. Assuming (8), we get $\Gamma_{ij}^{\alpha} = -\frac{\partial\varphi_\alpha}{\partial x^j}\delta_i^\alpha$, with non-zero $\Gamma_{\alpha j}^{\alpha} = \frac{\partial\varphi_\alpha}{\partial x^j}$. Substituting Γ_{ij}^{α} in (7) we obtain simple base equations

$$\frac{\partial Y_\alpha}{\partial x^i} - \frac{\partial\varphi_\alpha}{\partial x^i}Y^\alpha = 0, \qquad \dot{Y}^\alpha - \nu_\alpha Y^\alpha = 0, \qquad \nu_\alpha = (k_\alpha x - \omega_\alpha). \tag{9}$$

Since $x^0 = t$, $k_\alpha = (k_{\alpha 1},\ldots,k_{\alpha N})$, $\frac{\partial\varphi_\alpha}{\partial x^i} = k_{\alpha i}$, if $i \neq 0$, or $\frac{\partial\varphi_\alpha}{\partial x^0} = \frac{\partial\varphi_\alpha}{\partial t} = \omega_\alpha$ for the case of $i = 0$.

Using Gauges for the Neoclassical Economics

Identifying the utility with cumulate $\varphi_\alpha(x,t)$ of the growth rates $\dot{\varphi}_\alpha(x,\dot{x},t) = \nu_\alpha$ of the gross product Y^a in the industry α of (9), let's consider labor L, capital K and technological progress A as factor-productivities in a single industry model, with no direct relationship to time. Then the last equation in (9) can be rewritten as

$$\nu = k\dot{x}, \qquad \nu(x,\dot{x}) = (\nabla\varphi,\dot{x}) \tag{10}$$

where $x = (L,K,A)$ and k is the vector of marginal utilities for factors L,K,A.

Cobb-Douglas function. We assume $\nabla\varphi = k = \left(\frac{\alpha}{L},\frac{\beta}{K},\frac{1}{A}\right)$, to expresses the contention that the factor-productivities L,K,A obey the Ricardian Law of Diminishing Returns [2]. Following Solow [6] the technological progress is assumed a residual A weighted to unity in regards to the growth rates of the gross product explained through factor-productivities of labor and capital. According to (10) we obtain Cobb-Douglas formula mathematically, whereas they were found empirically [4]. This points at a deeper meaning of this formula as perhaps representing the fundamental nature of the Law of Diminishing Returns, which was used to derive formula and equations (11)

$$\nu_Y\left(K,L,A\right)=\alpha\frac{\dot{L}}{L}+\beta\frac{\dot{K}}{K}+\frac{\dot{A}}{A},\qquad \dot{Y}=\nu_Y Y,\qquad Y=AL^{\alpha}K^{\beta}\tag{11}$$

The Solow model adds to the Cobb-Douglas function yet another equation, namely, the capital self-reproduction $\dot{K}=sY-\delta K$ from where we get $\nu_K\left(K,Y\right)=s\frac{Y}{K}-\delta.$

The marginal utility of capital generation as a factor productivity $\frac{\partial\varphi_K}{\partial Y}=s\frac{d(lnK)}{d(lnY)}/\dot{K}.$

The marginal self-correction of capital as a factor productivity $\frac{\partial\varphi_K}{\partial K}=-\delta/\dot{K}.$

Accordingly the Solow model can be rewritten as

$$\dot{Y}=\nu_Y Y,\qquad \dot{K}=\nu_K K.\tag{12}$$

This brings us to equations tracing the phase gradients for factor productivities.

Gauges in the Monetary Economy, the "Invisible" Hand

The equations for self-regulating markets. We use equations based on (9)–(10), where Y-is the GDP, M is the monetary mass, which mutually interplay to achieve equilibrium

$$\dot{Y}=\nu_Y Y,\quad \dot{M}=\nu_M M,\quad \nu_Y=\frac{d\varphi_Y(Y,M)}{dt},\quad \nu_M=\frac{d\varphi_M(Y,M)}{dt}\tag{13}$$

Similar to the case of neoclassics, we build full derivatives of phases φ_Y, φ_M, using partial derivatives as in the second equation of (10) i.e. $(\nabla\varphi,\dot{x})$. Marginal utilities for factors Y and M are shown below [3].

$$\frac{\partial \varphi_Y}{\partial Y} = h\left(1 - \frac{Y}{CC}\right)/\dot{Y}$$ Marginal self-generation of the gross product assumed equal to diminishing returns per unit of growth

$$\frac{\partial \varphi_Y}{\partial M} = -i/\frac{\dot{M}}{M}$$ Marginal correction of the gross pro- -duct through the monetary mass, which are in inverse relationship to the latter's growth rates

$$\frac{\partial \varphi_M}{\partial Y} = \alpha/\frac{\dot{Y}}{Y}$$ Marginal generation of the monetary mass through the supply of the real product is inversely proportional to its growth rates

$$\frac{\partial \varphi_M}{\partial M} = -\varepsilon/\frac{\dot{M}}{M}$$ Marginal self-correction of the monetary mass inversely proportional to the growth rates.

Substituting marginal utilities expressed through phases φ_Y, φ_M above into full derivatives ν_Y, ν_M we obtain the equations of market's self-regulation (14) for $\dot{\varphi}_Y$, $\dot{\varphi}_M$

$$\dot{Y} = Y\left(h\left(1 - \frac{Y}{CC}\right) - iM\right), \quad \dot{M} = M\left(\alpha Y - \varepsilon M\right),$$

$$\nu_Y = h\left(1 - \frac{Y}{CC}\right) - iM, \quad \nu_M = \alpha Y - \varepsilon M. \quad (14)$$

Conclusion: by using gauges we demonstrated that self-regulation is a feature of a closed system. This creates a fundamental paradox since any closed system is devoid of sources of growth by definition.

References

[1] L. Badalian, V. Krivorotov. Technological Shift and the Rise of a New Finance System: The Market Pendulum Model. *EJESS*, 21(2):233–66, 2008.

[2] L. Badalian, V. Krivorotov. Looking for a Single Root-Cause of both Crises. A New Reading of the Ricardian Law of Diminishing Returns. *JIE*, 8(2):173-99, 2011.

[3] L. Badalian, V. Krivorotov. A Financialized Monetary Economy of Production by Andrea Fumagalli and Stefano Lucarelli: A Comment. *IJPE*, 41(1):95-107, 2012.

[4] C. Cobb, P. Douglas. Theory of Production. *The American Economic Review. Supplement, Papers and Proceedings*, 18(1):139-165, 1928.

[5] B. Dubrovin, A. Fomenko, S. Novikov. Modern Geometry. Methods and Applications. *Springer-Verlag, GTM 93, Part 1*, 1984.

[6] R. Solow. Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*, 39(3):312-320, 1957.

Ամբողջ թվի ներկայացումը իրարից տարբեր բնական թվերի m-րդ աստիճանների գումարների և տարբերությունների տեսքով

Ա.Ս. ՄԻՔԱՅԵԼՅԱՆ

Կրթության Ազգային Ինստիտուտ

Սույն աշխատանքը գրելու շարժառիթը «ՔՎԱՆՏ» ամսագրի թիվ 2328 խնդիրն է:

Խնդիր 1. *Հնարավոր է յուրաքանչյուր ամբողջ թիվ ներկայացնել ամբողջ թվերի խորանարդների գումարի տեսքով, որոնց մեջ չլինեն հավասար թվեր*:

Խնդիր 1-ի ընդհանրացվածը կլինի.

Խնդիր 1*. *Յուրաքանչյուր ամբողջ թիվ հնարավոր է ներկայացնել իրարից տարբեր բնական թվերի m-րդ աստիճանների գումարների և տարբերությունների տեսքով*:

Խնդիր 1.** *Յուրաքանչյուր ամբողջ թիվ հնարավոր է ներկայացնել իրարից տարբեր ամբողջ թվերի m-րդ աստիճանների գումարի տեսքով, որտեղ m-ը կենտ բնական թիվ է ($m = 3$ դեպքում կստանանք խնդիր 1-ը)*:

Խնդիր 1*-ի լուծումը ստացվում է հետևյալ փաստից.

Լեմմա. *Դիցուք m-ը 1-ից մեծ բնական թիվ է, իսկ c_0-ն այնպիսի բնական թիվ է, որ* (1) *հավասարումը ունի անվերջ թվով*

$$\varepsilon_1 x_1^m + \varepsilon_2 x_2^m + ... + \varepsilon_k x_k^m = c_0 \tag{1}$$

$(x_1, x_2, \ldots, x_k) = (a_{1n}, a_{2n}, ..., a_{kn})$ *լուծումներ, որտեղ* $\varepsilon_i \in \{-1; 1\}$, $k \in \mathbb{N}$, *իսկ* a_{in} $(i = 1, 2, \ldots, k)$, $n \in \mathbb{N}$ *բնական թվերը միմյանցից տարբեր են: Այդ դեպքում յուրաքանչյուր ամբողջ թիվ հնարավոր է ներկայացնել իրարից տարբեր բնական թվերի m-րդ աստիճանների գումարների ու տարբերությունների տեսքով: Ընդ որում, յուրաքանչյուր ամբողջ թվի համար այդպիսի ներկայացումներն անվերջ են*:

Սահմանում.

$$\Delta_m^s (x, p) = \Delta_m^{s-1} (x, p) - \Delta_m^{s-1} \left(x, p - 2^{s-1}\right),$$

որտեղ $1 \leq s \leq m$, $p \geq 2^{s-1}$, $m, p, s \in \mathbb{N}$, $x \in \mathbb{R}$:

$\Delta_m^s (x, p)$ երկու փոփոխականի ֆունկցիան կանվանենք 2^s հատ երկանդամների՝ $x + p$, $x + p - 1$, $\ldots$, $x + p - (2^s - 1)$ m-րդ աստիճանների տարբերությունների բազմանդամ: Այլ կերպ ասած՝ $\Delta_m^s (x, p)$-ը 2^s հաջորդական երկանդամների m-րդ աստիճանների՝ $(x + p)^m$, $(x + p - 1)^m$, $\ldots$, $(x + p -$

$2^{s-1})^m$, $(x+p-(2^{s-1}-1))^m$, $(x+p-(2^s-1))^m$ առաջին և երկրորդ 2^{s-1}-յակների նախորդ փարբերությունների փարբերությունն է:

$\Delta_m^s(x,p)$ բազմանդամի հիմնական հատկությունը

Հատկություն. $\Delta_m^m(x,p)=2^{\frac{m(m-1)}{2}}m!=c_0$:

Վերցնելով $x=n\in\mathbb{N}$ և $p=2^m-1$, կստանանք $k=2^m$ հաջորդական բնական թվերի m-րդ աստիճաններ՝ $x_1^m=(n+2^m-1)^m$, $x_2^m=(n+2^m-$ $-2)^m,\ldots,x_{k-2}^m=(n+2)^m$, $x_{k-1}^m=(n+1)^m$, $x_k^m=n^m$, որոնք կբավարարեն (1) հավասարմանը՝ $\varepsilon_1x_1^m+\varepsilon_2x_2^m+\cdots+\varepsilon_kx_k^m=\Delta_m^m(n,2^m-1)=2^{\frac{m(m-1)}{2}}m!=c_0,\forall n\in\mathbb{N}$: Համապատասխան $\varepsilon_i\in\{1;-1\}$, $i=1,2,\ldots,k=2^m$ կորոշվեն հետևյալ կարգով ($\varepsilon_1=1$ կամ $\varepsilon_1=-1$ կգրենք համապատասխանաբար «+» և «-» նշաններով):

Ընդհանրապես 2^m-յակի նշանները կորոշվեն $\Delta_m^m\to\left(\Delta_{m-1}^{m-1}\right)-\left(\Delta_{m-1}^{m-1}\right)$ անդրադարձ բանաձևով: Խնդիր 1*-ի լուծումը ավարտելու համար մնում է օգտվել լեմմայից: Սակայն խնդիր 1-ի լուծման համար որպես c_0 կարելի վերցնել ինչպես $c_0=2$, $k=3$, այնպես էլ՝ $c_0=1$, $k=3$: Առաջին դեպքում մենք կստանանք $x_1^3+x_2^3+x_3^3=x^3+y^3+z^3=2$ հավասարումը, որի համար իրարից փարբեր անվերջ թվով ամբողջ լուծումներ կլինեն, օրինակ՝ $x=6n^3+1$, $y=1-6n^3$, $z=-6n^2$ եռյակները, որտեղ $n\in\mathbb{Z}$ և $n\neq 0$: Երկրորդ դեպքում $x^3+y^3+z^3=1$ հավասարման համար՝ $x=9n^4$, $y=1-9n^3$, $z=3n-9n^4$, $n\neq 0$, $n\in\mathbb{Z}$ եռյակները:

Խնդիր 1-ը և Ուորինգի պրոբլեմը

1770թ.-ին անգլիացի մաթեմատիկոս Էդուարդ Ուորինգը (Վարինգ) արեց հետևյալ ենթադրությունը.

Յուրաքանչյուր $m>1$ բնական թվի համար գոյություն ունի այնպիսի $k=k(m)$ բնական թիվ, որի դեպքում $\forall M\in\mathbb{N}$ թիվ կարելի է ներկայացնել k հատ ոչ բացասական ամբողջ թվերի m-րդ աստիճանների գումարի տեսքով՝ $M=x_1^m+x_2^m+\cdots+x_k^m$, $x_i\in\mathbb{N}_0$:

Մասնավորապես Ուորինգի խնդրում $m=2$ դեպքում նվազագույն $k_0=k(2)=4$, ապա խնդիր 1*-ում՝ $m=2$ դեպքում նվազագույն $\widetilde{k_0}(2)=3$-ն է, իսկ $m=3$-ի համար $k_0(3)=9$, մինչդեռ խնդիր 1**-ում $m=3$-ի համար $\widetilde{k_0}(3)\leq 7$:

Խնդիր 2. *Յուրաքանչյուր ամբողջ թիվ հնարավոր է ներկայացնել 7 իրարից փարբեր ամբողջ թվերի խորանարդների գումարի տեսքով:*

$$s=s^3+\left(\frac{s-s^3}{6}+1\right)^3+\left(\frac{s-s^3}{6}-1\right)^3+\left(\frac{s-s^3}{6}\right)^3+$$
$$+\left(8\cdot\frac{s-s^3}{6}\right)^3+\left(9\cdot\frac{s-s^3}{6}\right)^3+\left(s-s^3\right)^3:$$

Երբ $s \in \{0; -1; 1\}$, ապա $s = s^3 + 2^3 + (-2)^3 + 3^3 + (-3)^3 + 4^3 + (-4)^3$: