FORMATION OF BELL STATES BY RANDOM MAPPINGS OF THE TWO-DIMENSIONAL FOCK STATE

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One of the important directions in the development of quantum communications is related to the problem of creating and controlling Bell's states. We propose a new way to create and control such states, based on quantum mechanics with a fundamental environment recently developed by the author [1].

Let us consider the *quantum subsystem+random environment* as a joint system (JS), which is described in the framework of the stochastic differential equation of the Langevin-Schrödinger type:

$$i\partial_t\Psi_{stc} = \hat{\mathrm{H}}\big(\mathbf{x}, t; \{\mathbf{f}(t)\}\big)\,\Psi_{stc}, \qquad \mathbf{x} = (x_1, x_2), \qquad \partial_t \equiv \partial/\partial t, \tag{1}$$

where the evolution operator $\hat{\mathrm{H}}\big(\mathbf{x}, t; \{\mathbf{f}(t)\}\big)$ of JS is represented as two linearly coupled $1D$ harmonic oscillators immersed in a random environment:

$$\hat{\mathrm{H}}\big(\mathbf{x}, t; \{\mathbf{f}(t)\}\big) = \frac{1}{2}\sum_{l=1}^{2}\Big[-\frac{\partial^2}{\partial x_l^2} + \Omega^2\big(t; \{\mathbf{f}(t)\}\big)x_l^2\Big] + \omega\big(t; \{\mathbf{f}(t)\}\big)x_1 x_2, \tag{2}$$

$x_1, x_2;\ t \in (-\infty, +\infty)$. Note that $\Omega\big(t; \{\mathbf{f}(t)\}\big)$ and $\omega\big(t; \{\mathbf{f}(t)\}\big)$ are some random functions. As for the $\{\mathbf{f}(t)\}$ function, this is a random Markov process, which turns on at the moment of time $t_0 > -\infty$ and turns off at the moment of time $t_0' < +\infty$. It is easy to show that for times $t < t_0$ equation (1)-(2) describes $2D$ Fock states.

Theorem. *If a complex probabilistic process* $\Psi_{stc}\big(\mathbf{x}, t; \{\mathbf{f}(t)\}\big)$ *satisfies the equation (1)-(2), and the random function* $\{\mathbf{f}(t)\} = \big(f^i(t), f^r(t)\big)$*, respectively, respectively, satisfies to the correlation relations of white noise*:

$$\langle f_l^{\upsilon}(t)\rangle = 0, \qquad \langle f_l^{\upsilon}(t) f_l^{\upsilon}(t')\rangle = 2\epsilon_l^{\upsilon}\delta(t - t'), \qquad \upsilon = (i, r), \tag{3}$$

where ϵ^{υ} *denotes the power of the random fluctuations, then the mathematical expectations of the Bell states can be constructed exactly in the form of multiple integrals and solutions of second-order partial differential equations (PDE):*

$$\bar{\Psi}^{\mp}(q_1, q_2, t) = \mathbb{E}\big[\Psi^{\mp}_{QSE}\big] = \frac{1}{\sqrt{2}}\big\{\overline{|0\rangle}_1 \otimes \overline{|0\rangle}_2 \mp \overline{|1\rangle}_1 \otimes \overline{|1\rangle}_2\big\},$$

$$\bar{\Phi}^{\mp}(q_1, q_2, t) = \mathbb{E}\big[\Phi^{\mp}_{QSE}\big] = \frac{1}{\sqrt{2}}\big\{\overline{|0\rangle}_1 \otimes \overline{|1\rangle}_2 \mp \overline{|1\rangle}_1 \otimes \overline{|0\rangle}_2\big\}, \tag{4}$$

where $q_1 = (x_1 - x_2)/\sqrt{2}$ and $q_2 = (x_1 + x_2)/\sqrt{2}$, in addition, $\bar{\Psi}^{\mp}_{QS}(q_1, q_2, t) = \langle \Psi^{\mp}_{QSE} \rangle_{\mathbb{R}_{\{\boldsymbol{\xi}\}}}$ and $\bar{\Phi}^{\mp}_{QS}(q_1, q_2, t) = \langle \Phi^{\mp}_{QSE} \rangle_{\mathbb{R}_{\{\boldsymbol{\xi}\}}}$ denote Bell states, which are obtained after averaging the corresponding expressions over the functional space $\mathbb{R}_{\{\boldsymbol{\xi}\}}$. As for the wave states $\overline{|0\rangle}_l$ and $\overline{|1\rangle}_l$, they are calculated and have the following form:

$$\begin{aligned}
\overline{|0\rangle}_l &= (g_l^-)^{1/2} \int_{-\infty}^{+\infty} \int_0^{+\infty} \bar{W}_l^{(\frac{1}{2},\frac{1}{2})}(u_1, u_2, t) \exp\Big\{\frac{1}{2}\big(iu_1 - u_2\big) q_l^2\Big\} du_1 du_2, \\
\overline{|1\rangle}_l &= 2(g_l^-)^{1/2} q_l \int_{-\infty}^{+\infty} \int_0^{+\infty} \bar{W}_l^{(\frac{3}{2},\frac{3}{2})}(u_1, u_2, t) \exp\Big\{\frac{1}{2}\big(iu_1 - u_2\big)\, q_l^2\Big\} du_1 du_2,
\end{aligned} \tag{5}$$

where $g_l^- = (\Omega_l^-/\pi)^{1/2}$ and $(\Omega_l^-)^2 = \lim_{t\to-\infty}\big[\Omega^2\big(t;\{\mathbf{f}(t)\}\big) - (-1)^l \omega\big(t;\{\mathbf{f}(t)\}\big)\big]$. In addition, the function $W_l^{(p,k)}$ is a solution of the following second-order PDE:

$$\partial_t W_l^{(p,k)} = \big\{\hat{L}_l - (pu_1 + iku_2)\big\} W_l^{(p,k)}. \tag{6}$$

where the operator $\hat{L}_l$ has the form:

$$\hat{L}_l = \epsilon_l \bigg(\frac{\partial^2}{\partial u_1^2} + \mu \frac{\partial^2}{\partial u_2^2}\bigg) + \frac{\partial}{\partial u_1}\big(u_1^2 - u_2^2 + \Omega_{0l}^2(t)\big) + 2u_1 \frac{\partial}{\partial u_2} u_2.$$

Recall that in the operator $\hat{L}_l$ the parameter $\mu = \epsilon_l^i/\epsilon_l^r \in [0, 1]$ denotes a constant, and $\Omega_{0l}^2(t) = \big[\Omega^2\big(t; 0\}\big) - (-1)^l \omega\big(t; 0\}\big)\big]$ is a certain regular function of time.

References

[1] Gevorkyan A. S. *Nonrelativistic quantum mechanics with fundamental environment, Theoretical Concepts of Quantum Mechanics.* **8**, 2012, 161–186. Ed. Prof. M. R. Pahlavani, ISBN: 978-953-51-0088-1.